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USE OF ECONOMETRIC METHODS IN FORECASTING MODELS--
WITH SPECIFIC REFERENCE TO THE WARP KNITTING INDUSTRY

A MASTER'S RESEARCH PAPER

Presented to

The Faculty of the Graduate Division

by

Wallace David Teich

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Industrial Engineering

Georgia Institute of Technology

February, 1966

USE OF ECONOMETRIC METHODS IN FORECASTING MODELS--

WITH SPECIFIC REFERENCE TO THE WARP KNITTING INDUSTRY

Approved:

Chairman

Date approved by Chairman: March 3, 1966

To my parents

ACKNOWLEDGMENTS

This study was originally planned as a Special Problem carrying three credit hours. However, as the work progressed, the author became increasingly interested in the subject of his research. In the opinion of his advisor, the results obtained were interesting enough to warrant their publication. As it was too late to negotiate a program change from "Non-Thesis" to "Thesis" designation, it was decided to submit this study in the form of a Master's Research Paper, so that a permanent record would be available for future reference.

The time, effort, direction, and patience provided by the author's advisor, Dr. Joseph Krol, during the preparation of this study were greatly appreciated. His suggestions and constructive criticisms helped make the text material more digestible. The personal interest in the author's academic activities, provided by Dr. Krol deserves special mention. The two Master's theses, cited in the Bibliography, were also directed by Dr. Krol.

Appreciation is also acknowledged to the members of the reading committee: Dr. Raul Gamoneda and Dr. Robert N. Lehrer, who provided the funds to have this study published.

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SUMMARY

The objectives of this study were to analyze some of the components of the demand for warp knitted fabrics and to forecast demand for this product during the years 1965, 1966, and 1967. With the aid of multiple regression and correlation analysis, three econometric models (Model I, Model II, and Model III) were developed. The basic equation for each model was:

$$Y_t = K\, X_1^{a+\alpha t}\, X_2^{b+\beta t}\, X_3^{c+\gamma t} \ldots X_n^{z+\zeta t}$$

where

Y_t is the dependent variable; i.e., shipments of warp knitted fabrics during the year t.

$X_1, X_2, X_3, \ldots, X_n$ are selected independent variables.

$K, a, \alpha, b, \beta, c, \gamma, \ldots, z, \zeta$ are numerical constants, and

t is time measured in years; i.e., in 1954 $t = 1$, in 1955 $t = 2$, etc.

The independent variables used in model development were as follows:

F = shipments of all knitted fabrics (in pounds) per capita,

I = disposable income (in current dollars) per capita, and

S = synthetic fiber production (in pounds) per capita.

Model I has the form:

$$Y_t = 501,800\ F^{1.386+0.001t}\ I^{0.008+0.001t}$$

Model II has the form:

$$Y_t = 2,531,000\ S^{0.118+0.052t}\ I^{-0.556-0.002t}$$

Model III has the form:

$$Y_t = 34,766,000\ F^{1.728-0.090t}\ I^{-0.779-0.027t}\ S^{-0.375+0.115t}$$

Statistical analysis of each model yielded the following results

1. a coefficient of multiple correlation greater than 0.99
2. a standard error of the estimate less than 0.025
3. a highly significant F ratio (from the analysis of variance table) indicating that variance was due to regression as opposed to deviation about the regression.

The validity of the predictions derived from each model depends upon the accuracy of the extrapolation of the observed data for each independent variable. It was found that the equations of the type

$$F_t = (1.4804)(1.0801)^t(1.040)^{t^2}$$

$$I_t = (1,873.4)(1.0243)^t(1.001)^{t^2}$$

$$S_t = (10.360)(1.0049)^t(1.007)^{t^2}$$

could be used as a feasible basis for short-range forecasts.

Forecasts for shipments of warp knitted fabrics were calculated for each model, using the extrapolated values of F_t, I_t, and S_t during the years 1965, 1966, and 1967.

The predictions were as follows:

Year	Model I	Model II	Model III	Mean
1965	227,550	233,100	228,430	229,690
1966	273,520	292,360	288,320	284,730
1967	332,400	374,580	368,440	358,440

It can be seen that Models I, II, and III, resulted in the "pessimistic," "optimistic," and "in-between" forecasts, respectively. Model III, based on three independent variables, produced forecasts very close to the mean value of the three forecasts.

CHAPTER I

INTRODUCTION

Forecasting is a science or an art of prediction, depending upon the methodology employed. To the housewife who prepares a weekly shopping list it is an art, but to the mathematically oriented econometrician forecasting is a science. The goal of any successful forecast is to reduce the area of avoidable risk. The techniques of forecasting, according to Roos (1) are divided into five classes:

1. Naive--"unsophisticated, scientifically uninstructed projections," such as linear projections, guessing, coin-tossing, and subjective extrapolations.

2. Leading indexes--the use of indicators which usually or always change prior to a change in the index (or variable) to be forecasted; e.g., shipments of goods always precede earnings.

3. Comparative pressures--involve ratios or differences in amplitude of one variable with respect to another. These ratios, such as inventory to sales, production to capacity, shipments to new orders, are used as guides in formulating a forecast.

4. Opinion polls--this subjective method of forecasting is derived from weighted of unweighted averages of the guesses of people who have some insight and/or information with respect to the variables being forecasted.

5. Econometrics--forecasting by means of the union of economic

theory with mathematics, statistics, and accounting.

Use of econometric methods involve the development of econometric models which are equations or systems of equations that will show how economic forces (or variables) operate. With the increased use of electronic digital computers that are capable of solving complex equations employing numerous variables, this method of forecasting has witnessed increased popularity on the part of the decision maker. An econometric approach to mathematical model building should pattern the following elements (2):

1. Development of a satisfactory theory or theories to explain [cause-and-effect] relationships between economic forces.
2. Translation of the theory or theories into mathematical equation using symbols.
3. Calculation of the constants in the equation based upon past data.
4. Using the model to forecast future values of econometric measures.
5. Evaluating the usefulness (reliability and validity) of the model for explaining economic relationships and forecasting the values of econometric measures.

Some of the econometric models recently developed have their foundations based upon sophisticated mathematical theory, the understanding, interpretation and application of which often exceeds the comprehension of the "average reader." Sir Dennis Robertson examines the spectrum of econometrics from a humorous point of view in his poem, "The Non-Econometrician's Lament" (see Appendix I).

This study will be concerned with analyzing and forecasting demand. The term *demand* may be defined as the consumption of a product or service which will be a function of a set of economic forces, such as price, income, population changes, substitute products, credit conditions, etc. Properly constructed equations will provide the user with a tool for forecasting demand and a means for understanding and controlling the interactions among these economic forces.

There are two classifications of forecasting: short-term forecasting and long-term forecasting. Short-term forecasting spans a duration of not more than 12 months (3). Within this period it is possible to observe the influence of such factors as advertising, sales campaigns, seasonal variations, and weather, upon the consumption of a product or service. Data for this type of forecast are usually gathered by an opinion poll.

Long-term forecasts normally involve a more detailed analysis and span a period of three or more years, with forecasts made periodically (4). The method of forecasting employed and the nature of the data assembled should be a function of the ultimate intention of the forecast.

How accurate the forecast will be, according to Barish (5), will depend upon the following factors:

1. adequacy and completeness of the data analyzed,
2. quality of the forecaster's judgment,
3. variability of sales patterns in the company and industry,
4. the time and money spent on the forecast, and
5. the nature of the product and the nature of the demand and market for it.

The objective of this study will be the development of econometric models, based on historical data, to analyze and forecast the demand for warp knitted fabrics. Predictions will be made for the years 1965, 1966, and 1967.

CHAPTER II

BASIC FACTS PERTAINING TO THE KNITTING INDUSTRY

The knitting industry is a subset of the textile industry, as shown in Figure 1. All machine-knitted fabrics are either weft knitted or warp knitted.

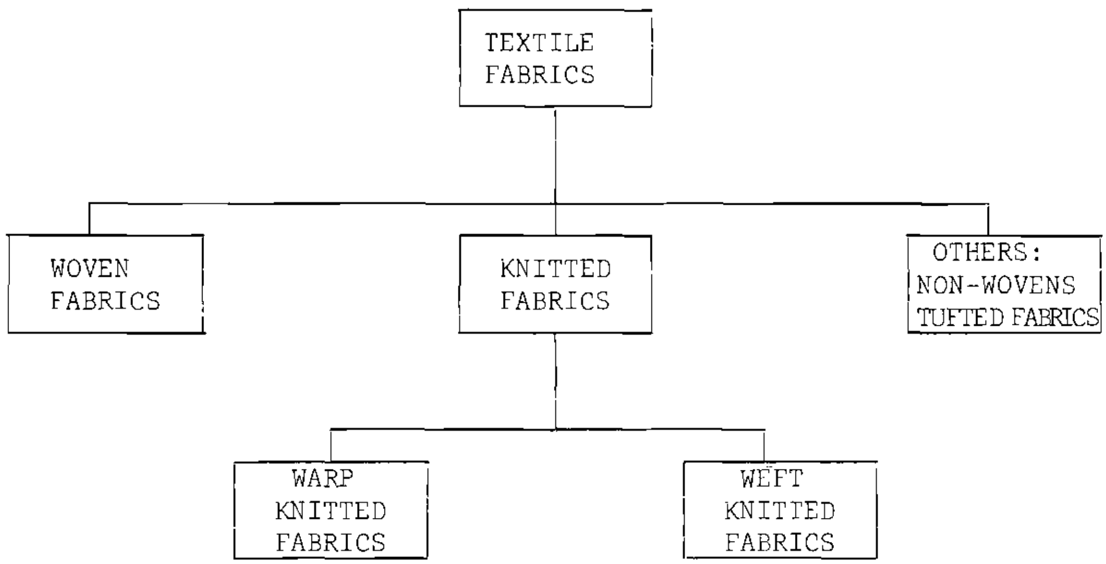

Figure 1. Subsets of the Textile Industry.

A weft knitted fabric is characterized by a yarn that forms loops in a horizontal direction. There are three varieties of weft knitted fabrics, each of which is manufactured on a different type of machine: (a) socks and stockings, (b) fabrics that are knit to fit (e.g., a full fashioned sweater), and (c) fabrics that are to be cut and sewn into garments (e.g., men's undergarments). Nearly all sweaters are weft knitted.

A warp knitted fabric forms loops that interlace with one another in a lengthwise direction. When compared (dimensionally) to a weft knit, warp knitted fabrics appear to have a closer stitch formation and a flatter and less porous appearance. A smoother, silkier cloth is produced that has relatively little bulk. These characteristics make this fabric ideal for women's nightwear and lingerie. Warp knitted fabrics may be produced with yarn that will not run and designed to incorporate a variety of mesh and openwork designs (i.e., designs with holes), such as laundry bags, fish nets, industrial filters and lace fabrics.

A better understanding of the uses and applications of warp knitted fabrics may be gained upon examination of Figure 2. The number to the right of each product classification represents the approximate share of the market (within the realm of the warp knitting industry) that each component fabric has.

In this study, warp knitted fabrics will be classified as a consumer semi-durable commodity; i.e., its estimated life cycle varies from one to five years. When dealing with most semi-durable products, it is often impractical to apply mathematical techniques, as unexpected demand fluctuations stemming from fashion or style changes (not accounted for in the mathematical model) may lead to distorted forecasts. Merrett and Bannock (6) state that "light consumer goods" or semi-durable goods differ from consumer durable goods in several respects:

1. they absorb a smaller portion of the consumers' disposable income,

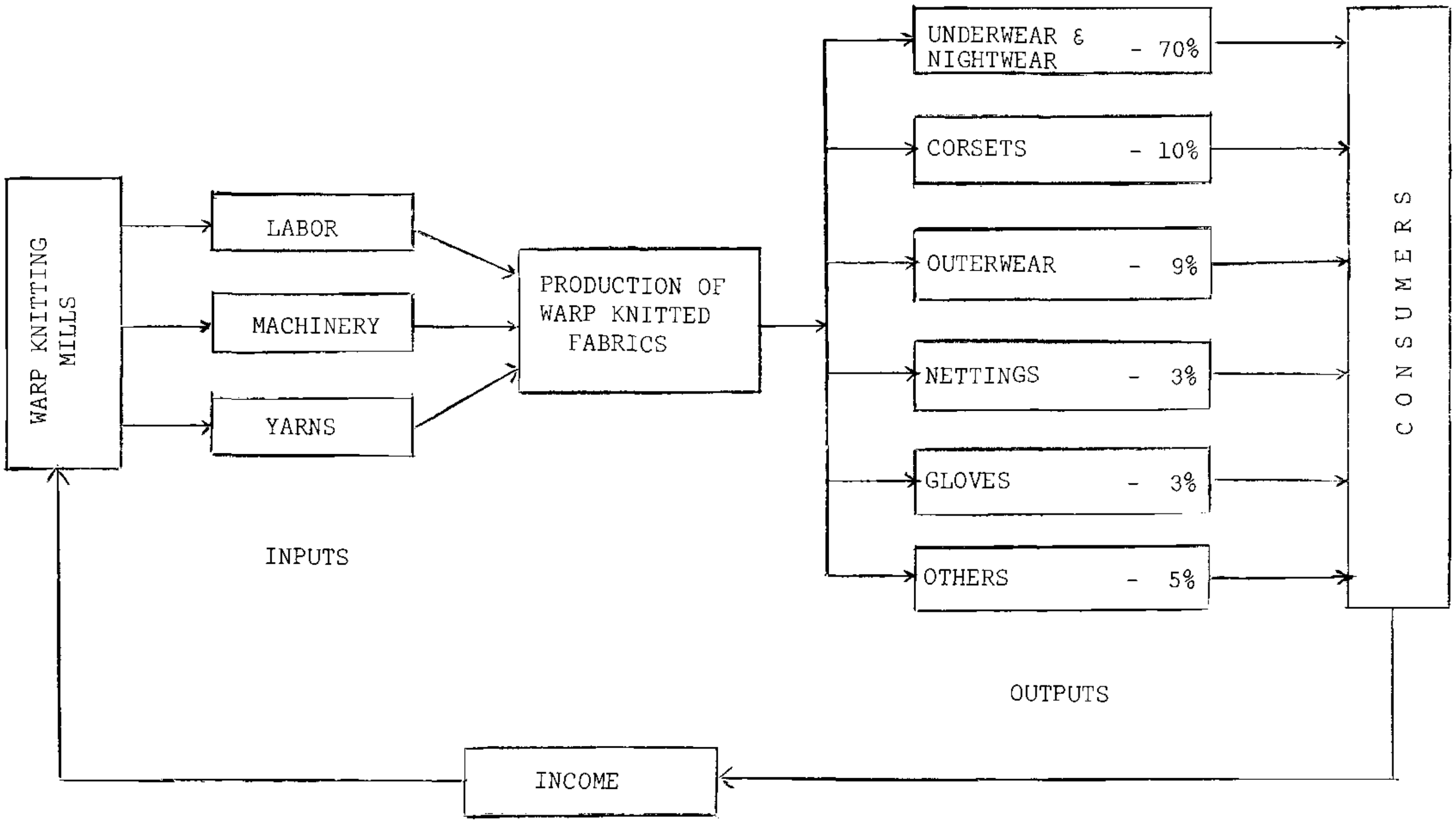

Figure 2. Input-Output Diagram of the Warp Knitting Industry.

2. they are purchased at more frequent intervals, and

3. they are more susceptible to sales pressure, fashion, and habit than consumer durables.

In this study, it will be assumed that any fluctuations in demand, arising from such variables as fashion, advertising, and similar factors, will be randomized and accounted for within the econometric model. To support the assumption the following reasons are given:

1. The product to be analyzed is a fabric as opposed to a garment. Style changes in apparel will require fabrics of a comparable nature.

2. Over 90 per cent of all warp knitted fabrics are of one basic style or fabric construction called "Jersey," which is primarily used in nightwear and lingerie.

3. Within the three-year period involved in the forecast, the probability of a new product being put on the consumer market that will have a substantial impact on the demand for warp knitted fabrics is very small.

4. During the past 18 years the share of warp knitted fabrics within the total knitted market has risen from 30 per cent to 40 per cent (see Table 10).

5. Assignable causes which can explain this rise in demand are as follows:

   a. new product developments; e.g., Tricot shirtings, laminated fabrics, and uniform fabrics,

   b. improvements within the two primary yarns used (i.e., nylon and acetate) as to strength, dye affinity, versatility, etc.;

c. improved machinery with respect to production and design versatility; and

d. increased awareness of "Tricot" (the trade name for warp knitted fabrics) on the part of the consumer.

> Tricot [warp] knitted fabrics which formerly depended on loungewear and lingerie markets are now being used in growing volume in sleepwear, rainwear, men's shirts, jackets, shoes, and industrial application. Replacement of some woven goods is seen because of economic advantage in increasing versatility and expanding types of fibers (7).

Merrett and Bannock further relate (8) that "once a product has developed stability in the market," meaning that demand has reached a specified level, "sales promotion and advertising will maintain a fixed ratio." Thus, "as a result of these stabilizing influences, it is possible to predict the total demand for two or three years ahead by extrapolation." In this study, econometric analysis will be employed instead of subjective extrapolation which is a naive method of forecasting.

CHAPTER III

SURVEY OF ECONOMETRIC METHODS INVOLVING DEMAND FUNCTIONS

During the past 20 years there have been many attempts to employ historical data in the development of mathematical models to forecast future happenings. Models have been developed for the national economy, for an industry and for individual organizations. The selective literature search is designed to explore the types of demand models used within an industry and to cite some of the techniques used to forecast demand in the textile industry, specifically the warp knitting industry.

The first significant study of the use of econometric methods to forecast demand was made by Roos and von Szeliski (9) in 1939. Employing regression analysis, models were developed relating the consumption of automobiles to the following independent variables:

1. supernumerary income [i.e., the difference between national income and living costs],
2. number of households,
3. consumers' stock of automobiles and their age distributions,
4. prices of cars, and
5. replacement pressure.

Later studies by Suits (10) and Chow (11), each using regression analysis, explored alternate procedures (and different independent variables) to forecast consumer demand for new automobiles. Chow outlined a clear procedure of the use of linear and multiplicative models. Suits reviewed the previous work in this field and recommended improved methods to calculate demand.

A more general model, using a system of simultaneous equations, was developed by Smith (12) to calculate the demand for all durable and non-durable goods. Although Smith's study is not related to any particular industry, the technique of manipulating systems of equations as a forecasting tool can easily be applied to any industry.

Miller (13) developed a mathematical model to forecast demand for refrigerators. By adapting this model to the consumer market, Miller was able to estimate the influence of income on the aggregate demand for refrigerators.

Rama Sastri (14) developed a dynamic model for the unit air-conditioner industry in the United States. This model which is applicable to any durable consumer product, defined new owner demand and replacement demand as a dynamic function of the demand for unit air-conditioners.

Wilson (15) treated carpeting (both tufted and woven) as a consumer durable product and using regression analysis developed equations to forecast long-term (up to four years) and short-term (up to six months) shipments of carpeting. The following independent variables were considered:

1. new housing starts (in dollars),
2. disposable personal income per household (in dollars),
3. commercial construction (in dollars),
4. commercial aircraft shipments (in number of aircraft), and
5. factory sales of cars sold (in dollars).

Disposable income and factory sales of automobiles were found to be statistically significant at the 0.001 and 0.10 probability level, respectively.

Ferguson and Polasek (16) developed econometric models where consumption of raw apparel wool in the United States was treated as a price-elastic commodity. A model which provides an indication of price and income elasticity for any textile product is of importance in determining demand patterns.

A similar study discussed the consumption per capita of wool in the European Common Market countries consisting of France, Italy, West Germany, Belgium, and the Netherlands (17). The factors considered as affecting the size of the market for this commodity were:

1. demand for woolen clothing by men and women,
2. variations in the age and sex composition of the population,
3. population changes,
4. income levels, and
5. clothing prices as indicated by wool prices.

Although this study did not emphasize a statistical approach, coefficients for price elasticity (0.3) and income elasticity (0.5) were obtained for the consumption of wool. Projections for expected demand for wool were calculated through 1975 during five-year intervals.

Choudhry (18) traced the history of the demand for burlap, by interpolation, and attempted to isolate any factors which will create demand for this fabric. The interrelationships studied were proven to be statistically insignificant.

Using the naive method of forecasting, previous described by Roos, there have been two estimates of shipments of warp knitted fabrics by individuals closely associated with the textile industry. How these estimates (or forecasts) were obtained is not stated, and it must, therefore, be assumed that they were derived subjectively with information being accumulated from a knowledge of market conditions, previous data, and new developments. The Director of the Tricot Institute of America states that demand "was" 180 million pounds in 1964 (19). There is approximately an 18-month lag until data are obtained. With more machines operating in 1965, it is stated that shipments "should substantially increase." The Manager of Product Information of Chemstrand, estimated shipments of warp knitted fabrics in 1964 at 172 million pounds (20). Observation of the data in Table 1 shows that shipments of warp knitted fabrics in 1964 were 197 million pounds.

CHAPTER IV

SELECTION OF AN EQUATION APPROPRIATE TO THE DATA AVAILABLE

The development of econometric models involves the selection of an equation, or system of equations, which in the judgment of the analyst is most likely to resemble the data assembled. Five possible equation types were considered in this study:

1. Linear
2. Polynomial
3. Sinusoidal
4. Logistic
5. Exponential

If the first differences of a variable are constant, the equation will be linear ($Y = ax + b$) and if the second, third, fourth, etc., differences are constant, a polynomial equation of the order of the difference will best fit the data. Observation of Table 1 shows that neither the first, second, third nor fourth differences are constant. Therefore, linear and polynomial equations were eliminated. Demand for warp knitted fabrics during the years 1954 through 1964 (Y in Table 1) follows an upward trend. No cyclical effects can be noticed and consequently a sinusoidal function was eliminated.

If the first or second differences of the logarithm (the base 10 was used throughout this study) of the variable increase by a constant, the data will closely follow a logistic function. Observation of the

Table 1. Differencing Demand for the Warp Knitting Industry

Year	Y*	ΔY	$\Delta^2 Y$	$\Delta^3 Y$	$\Delta^4 Y$
1954	60,097	8,198	-5,584	5,066	5,905
1955	68,295	2,614	-518	10,971	-14,963
1956	70,909	2,096	10,453	-3,992	-18,643
1957	73,005	12,549	6,461	-22,635	40,154
1958	85,554	19,010	-16,174	17,519	1,312
1959	104,564	2,836	1,345	18,831	-34,626
1960	107,400	4,181	20,176	-15,795	15,358
1961	111,581	24,357	4,381	-437	
1962	135,938	28,738	3,944		
1963	164,676	32,682			
1964	197,358				

* Demand is expressed in thousands of pounds.

data shown in Table 2 does not suggest a definable pattern. If the first or second differences of the logarithm are approximately constant, an exponential equation may be adapted to fit the data. The range of the first differences (Δ Log Y), in Table 2, is 0.08310 which is proportionately smaller than the other ranges calculated. Thus, an exponential equation will be used in this study.

Table 2. Logarithm Differencing of Demand for the Warp Knitting Industry

Year	Log Y	Δ Log Y	Δ^2Log Y
1954	4.77885	0.05654	-0.04023
1955	4.83439	0.01631	-0.01365
1956	4.85070	0.00266	-0.07622
1957	4.85336	0.07888	0.00824
1958	4.93224	0.08712	-0.07548
1959	5.01936	0.01164	0.00495
1960	5.03100	0.01659	0.06917
1961	5.04759	0.08576	-0.00249
1962	5.13335	0.08327	-0.00463
1963	5.21662	0.07864	
1964	5.29526		

In order to show changes in the dependent variable as a function of time, it is desirable to include time as a separate parameter in the equation. A dynamic equation was established to identify any fluctuations in demand for warp knitted fabrics. The resulting mathematical model is as follows:

$$Y_t = KX_1^{a+\alpha t}\ X_2^{b+\beta\varepsilon}\ X_3^{c+\gamma t}\ \ldots\ X_n^{z+\zeta t} \qquad \text{(Eq. 1)}$$

where:

Y_t is the dependent variable,

X_1, X_2, X_3, ... X_n are selected independent variables,

K, a, α, b, β, c, γ, ..., z, ζ are numerical constants, and

t is time measured in a selected unit.

This model is not restricted to the warp knitting industry, but may be used to forecast and analyze demand for many other products which exhibit similar patterns of behavior.

CHAPTER V

DEVELOPMENT OF ECONOMETRIC MODELS FOR THE WARP KNITTING INDUSTRY

This chapter contains the bulk of the independent research pertaining to this study. As it is considerably longer than the previous chapters, the material to be discussed will be subdivided into five sections; namely: Procedure for Extrapolating Independent Variables, Use of Computer Routine in Model Development, Model I, Model II, and Model III.

Procedure for Extrapolating Independent Variables

The independent variables selected for use in the development of the econometric models discussed in this chapter were as follows:

1. F = shipments of all knitted fabrics (in pounds) per capita,
2. I = disposable income (in current dollars) per capita, and
3. S = synthetic fiber production (in pounds) per capita.

Data for independent variables F, I, and S were available through the year 1964. To obtain data during the prediction periods for 1965, 1966, and 1967, it was necessary to generate future values. The least squares method of extrapolation was employed. Using the differencing procedures described in Chapter IV for the dependent variable, it was concluded that the second differences of the logarithm for each independent variable exhibited the least variability. On this basis it was decided that each independent variable, expressed as a function of time, could be written as:

$$X = uv^{t} w^{t^2} \quad \text{(Eq. 2)}$$

where

X is the selected independent variable,

t is time in years, and

u, v, and w are numerical constants determined by the method of least squares.

In order to determine the parameters u, v, and w, logarithms of both sides of Equation 2 were taken as follows:

$$\text{Log } X = \text{Log } u + t \text{ Log } v + t^2 \text{ Log } w \quad \text{(Eq. 2a)}$$

To facilitate calculations, Equation 2a was written as:

$$\text{Log } X = a + bt + ct^2 \quad \text{(Eq. 3)}$$

where

Log X is the logarithm of the selected independent variable (i.e., F, I, or S),

t is time in years,

a = Log u, b = Log v, and c = Log w.

The application of the method of least squares to Equation 3 yielded the following normal equations:

$$\left.\begin{aligned} \Sigma(\text{Log } X) &= Na + b\Sigma(t) + c\Sigma(t^2) \\ \Sigma(t \text{ Log } X) &= a\Sigma(t) + b\Sigma(t^2) + c\Sigma(t^3) \\ \Sigma(t^2 \text{ Log } X) &= a\Sigma(t^2) + b\Sigma(t^3) + c\Sigma(t^4) \end{aligned}\right\} \qquad \text{(Eq. 3a)}$$

The data for Log X was centered with $\Sigma t = 0$. The use of the centering method simplified the simultaneous equations as follows:

$$\left.\begin{aligned} \Sigma(\text{Log } X) &= Na + c\Sigma(t^2) \\ \Sigma(t \text{ Log } X) &= b\Sigma(t^2) \\ \Sigma(t^2 \text{ Log } X) &= a\Sigma(t^2) + c\Sigma(t^4) \end{aligned}\right\} \qquad \text{(Eq. 3b)}$$

To solve for numerical constants a, b, and c, the following equations were used:

$$b = \frac{\Sigma(t \text{ Log } X)}{\Sigma(t^2)} \qquad \text{(Eq. 3c)}$$

$$c = \frac{N\Sigma(t^2 \text{ Log } X) - \Sigma(t^2)\ \Sigma(\text{Log } X)}{N\Sigma(t^2) - \Sigma(X^2)^2} \qquad \text{(Eq. 3d)}$$

$$a = \frac{\Sigma(\text{Log } X) - c\Sigma(t^2)}{N} \qquad \text{(Eq. 3e)}$$

The general tabular formats which facilitated the calculations of the equations for Log F, Log I, and Log S, are shown in Tables 12, 13, and 14, respectively. The extrapolating equations developed were as follows:

$$\text{Log } F_t = 0.17042 + 0.0336t + 0.0017t^2 \qquad \text{(Eq. 4)}$$

$$\text{Log } I_t = 3.27406 + 0.0143t + 0.0001t^2 \qquad \text{(Eq. 5)}$$

$$\text{Log } S_t = 1.01538 + 0.0213t + 0.0030t^2 \qquad \text{(Eq. 6)}$$

(all with t = 0 in 1959)

Using the form given by Equation 2, Equations 4 through 6 may be rewritten as follows:

$$F_t = (1.4804)(1.0801)^t(1.040)^{t^2} \qquad \text{(Eq. 7)}$$

$$I_t = (1{,}873.4)(1.0243)^t(1.001)^{t^2} \qquad \text{(Eq. 8)}$$

$$S_t = (10.360)(1.0049)^t(1.007)^{t^2} \qquad \text{(Eq. 9)}$$

(all with t = 0 in 1959)

Figures 3, 4, and 5, show observed, calculated and extrapolated values for independent variables F_t, I_t, and S_t, respectively.

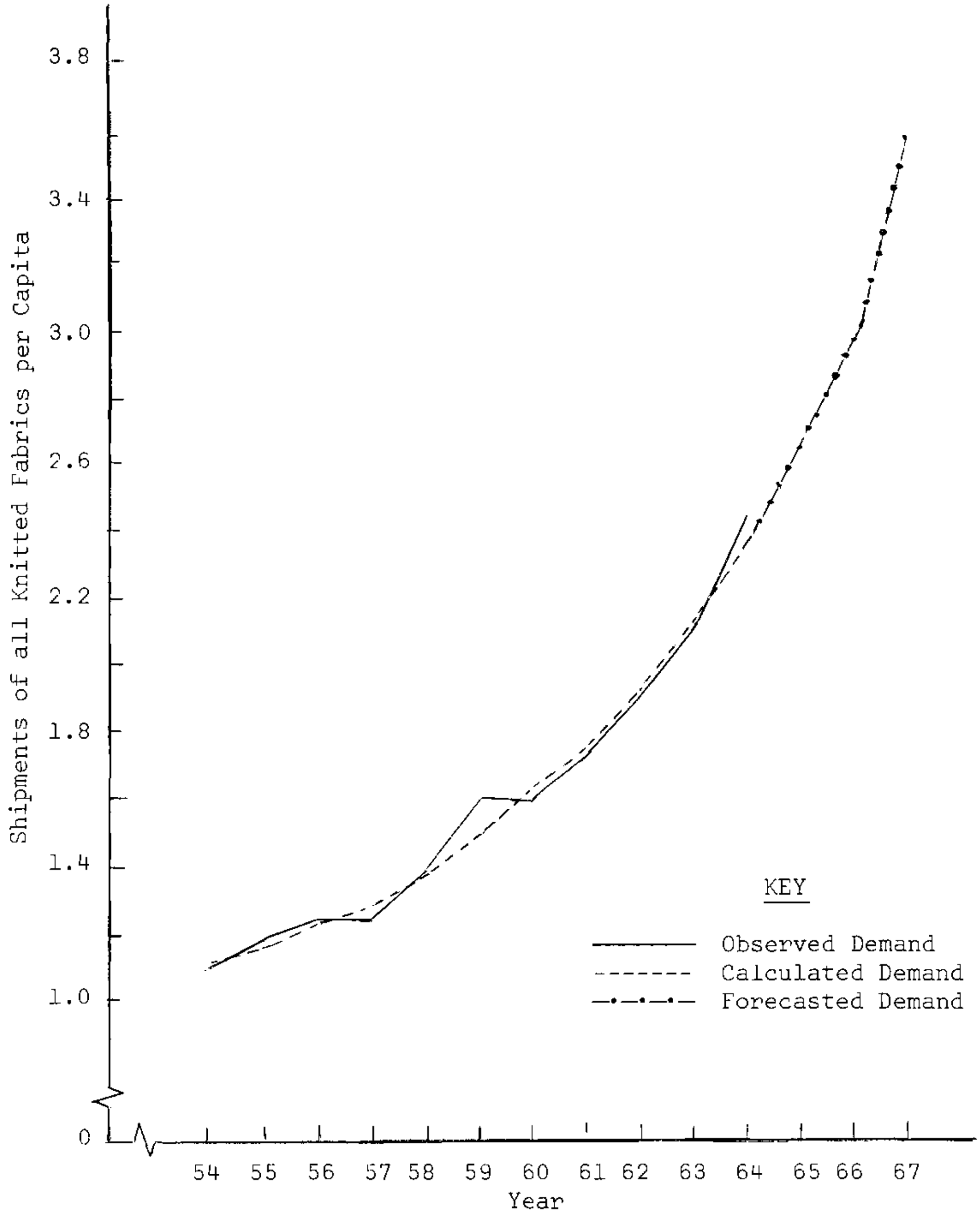

Figure 3. Extrapolation of Independent Variable F_t.

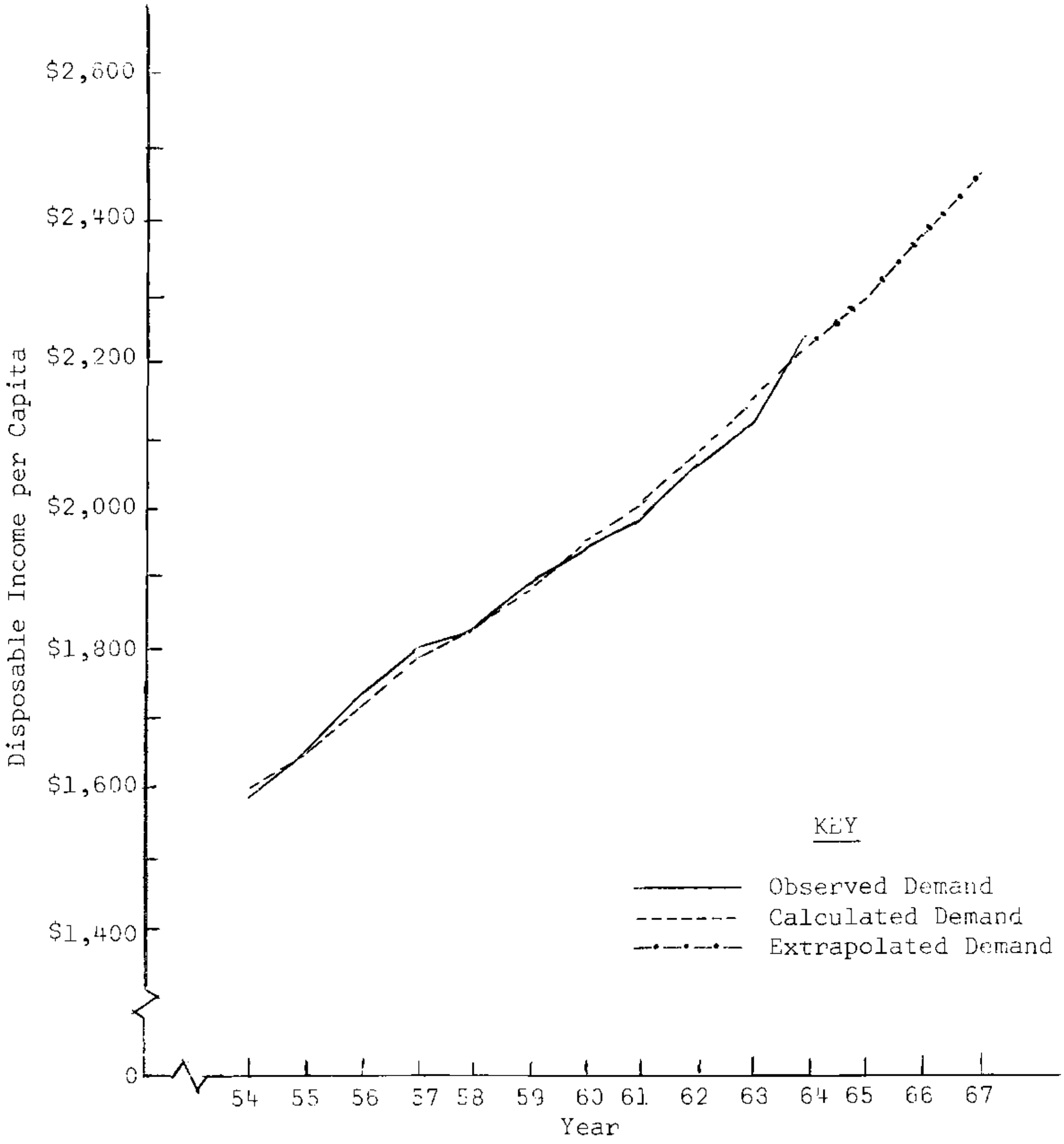

Figure 4. Extrapolation of Independent Variable I_t.

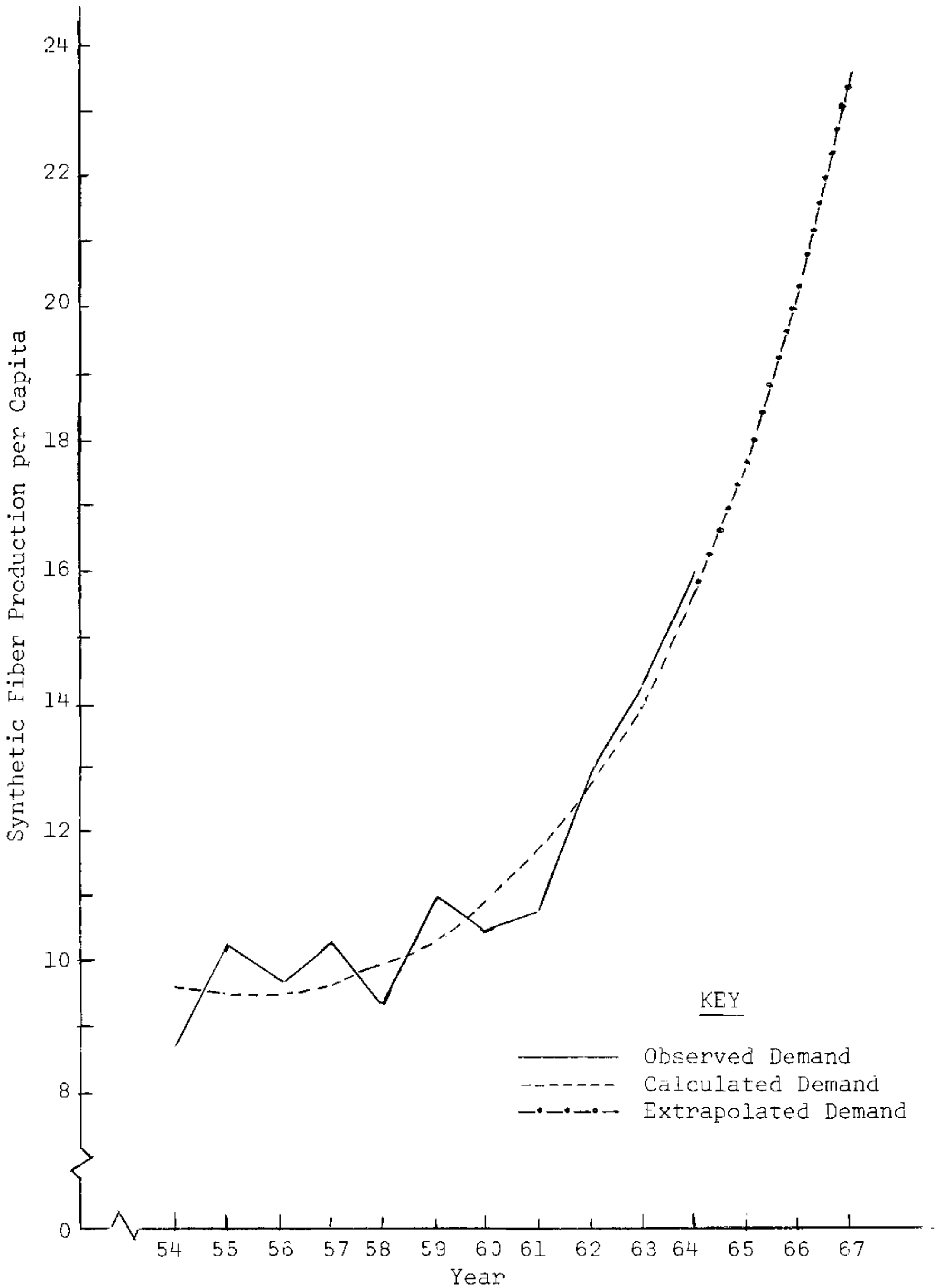

Figure 5. Extrapolation of Independent Variable S_t.

Use of Computer Routine in Model Development

The statistical technique of multiple regression and correlation analysis was used to determine numerical constants for each econometric model. To facilitate the mechanics involved, a computer routine was obtained from the program library of the Rich Electronic Computer Center. This routine entitled "General Multiple Regression and Correlation Analysis" (Number 5111) was designed by Charles L. Clark of the Burroughs Corporation (see Appendix III) for use on the B5500 electronic digital computer.

The above program requires that data be fed into the computer as a multiple linear equation in the form:

$$Y = K + aX_1 + bX_2 + cX_3 + \dots + zX_n \qquad \text{(Eq. 10)}$$

where

Y is the dependent variable,

$X_1, X_2, X_3, \dots, X_n$ are independent variables, and

K, a, b, c,...,z are numerical constants.

Referring to the mathematical model explained in Chapter IV (Eq. 1) the following steps were taken to obtain linearity:

$$Y_t = K\ X_1^{a+\alpha t}\ X_2^{b+\beta t}\ \dots\ X_3^{z+\zeta t} \qquad \text{(Eq. 11)}$$

$$\text{Log } Y_t = \text{Log } K + (a+\alpha t) \text{ Log } X_1 + (b+\beta t) \text{ Log } X_2 + \dots \qquad \text{(Eq. 12)}$$

$$\dots + (z+\zeta t) \text{ Log } X_n$$

$$\text{Log } Y_t = \text{Log } K + a \text{ Log } X_1 + \alpha t \text{ Log } X_1 + b \text{ Log } X_2 \qquad \text{(Eq. 13)}$$

$$+ \beta t \text{ Log } X_2 + \ldots + z \text{ Log } X_n + \zeta t \text{ Log } X_n$$

Once the program routine and data have been fed into the computer the resulting print-out will yield the following information:

1. all necessary numerical constants,
2. the calculated value (from the model) of each independent variable and its residual,
3. the coefficient of multiple correlation,
4. the coefficient of determination,
5. the standard error of the estimate,
6. the standard deviation of each regression coefficient,
7. analysis of variance for the multiple linear regression, and
8. partial correlation coefficients of each independent variable.

The discussion of each econometric model will pattern the following equation type:

$$Y_t = K\, X_1^{\,a}\, X_1^{\,\alpha t}\, X_2^{\,b}\, X_2^{\,\beta t} \ldots X_n^{\,z}\, X_n^{\,\zeta t} \qquad \text{(Eq. 14)}$$

This equation will facilitate the explanation of partial correlation coefficients and standard deviations of the regression coefficients.

Model I

This model has the form:

$$Y_t = K\, F^{a+\alpha t}\, I^{b+\beta t} \qquad \text{(Eq. 15)}$$

where

Y_t is shipments of warp knitted fabrics in thousands of pounds,

F is shipments of all knitted fabrics per capita,

I is disposable income per capita,

K, a, α, b, and β are numerical constants, and

t is time in years (with t = 0 in 1953).

Data for shipments of warp knitted fabrics were obtained from a publication of the U. S. Department of Commerce (21), as were data for shipments of all knitted fabrics. Represented are all commercial sales of finished fabrics and greige fabrics. Greige fabrics are fabrics as they leave the knitting machine (or weaving loom); i.e., they have not been scoured, bleached, dyed, or finished.

Not included in the data are knitted fabrics manufactured in a mill and processed into a garment in the same mill or another manufacturing facility of the same company; i.e., a vertically integrated organization.

Disposable income is defined as personal income earned less personal income taxes. It is the income the consumer has available to spend on commodities and services. Disposable income is measured in current dollars since the inflationary forces are measured by the numerical constants associated with the variable time.

The exponential function ($a+\alpha t$, $b+\beta t$, etc.) of each independent variable can be interpreted as an indication of elasticity. A positive exponential function infers that the independent variable is inelastic because any change in it will result in a variation of the dependent variable in the same direction. A negative exponential function indicates that the independent variable is elastic because any change in it will result in a variation of the dependent variable in the opposite direction. Unitary elasticity exists when the numerical value of the exponential function of the independent variable is equal to unity. In the econometric model to be developed, the value of the time variable was set equal to one in 1954, two in 1955, etc., so that with each succeeding year the exponential function develops increased weight. In 1965 time, t, will equal 12.

The econometric model resulting from the use of data for shipments of all knitted fabrics (in pounds) per capita (F), and for disposable income per capita (I), was as follows:

$$Y_t = 501{,}800\ F^{1.386+0.001t}\ I^{0.008+0.001t} \quad . \qquad \text{(Eq. 16)}$$

All statistical information concerning this model is shown in Table 4. Judging by the high coefficient of multiple correlation (0.997), low standard error of the estimate (0.014), and highly significant F ratio (from the analysis of variance table), this model is indicative of the data for shipments of warp knitted fabrics. Examination of the partial coefficients of correlation shows the data for F^a or $F^{1.386}$ to be a good indicator of the data for the dependent variable as its partial correlation coefficient is 0.78, whereas the same coefficients for $F^{\alpha t}$, I^b, and $I^{\beta t}$, are all positive and less than 0.10.

Disposable income per capita exerts little influence in the calculation of the dependent variable as illustrated by the low magnitude of its associated exponential function. Examination of the method used in calculating the forecasted values of the dependent variable (see Table 15), reinforces this fact. The small amount of weight possessed by $F^{\alpha t}$ in this model can be readily noted when F and I, shipments of warp knitted fabrics, are highly inelastic and disposable income is slightly inelastic.

To forecast demand for shipments of warp knitted fabrics during the years 1965, 1966, and 1967, the values obtained from the extrapolating equations for Log F (Eq. 4) and Log I (Eq. 5) were incorporated into the econometric model (Eq. 16) for each predicted year. The results calculated are shown in Table 3. It can be seen that each variable is expected to increase substantially during the prediction interval.

Table 3. Extrapolated Forecast of Demand for Model I

Year	F	I	Y_t
1965	2.70	$2,301	227,550
1966	3.06	2,382	273,520
1967	3.50	2,467	332,400

The procedure for calculating the extrapolated values to forecast Y_t is shown in Table 15 in Appendix II.

A graphical presentation of observed, calculated, and forecasted demand for shipments of warp knitted fabrics, using this econometric model, may be seen in Figure 6.

Table 4. Statistical Analysis of Model I

Year	t	Observed Demand (Y)	Calculated Demand (Y_t)	Residual ($Y-Y_t$)
1954	1	60,097	60,190	- 93
1955	2	68,295	67,990	+ 305
1956	3	70,909	71,860	- 951
1957	4	73,005	72,860	+ 145
1958	5	85,554	86,010	- 456
1959	6	104,564	103,500	+ 1,064
1960	7	107,400	103,370	+ 4,030
1961	8	111,581	118,290	- 6,709
1962	9	135,938	136,130	- 192
1963	10	164,676	159,800	+ 4,876
1964	11	197,358	199,410	- 2,052
				- 33*

Econometric Model	$Y_t = 501{,}800\ F^{1.386+0.001t}\ I^{0.008+0.001t}$	
Standard Error of Estimate	0.01416	
Coefficient of Multiple Correlation	0.9978	
Coefficient of Determination	0.9957	
Partial Correlation Coefficients	$F^a = 0.78$	$I^b = 0.01$
	$F^{\alpha t} = 0.02$	$I^{\beta t} = 0.06$
Standard Deviation of Regression Coefficients	$F^a = 0.46$	$I^b = 0.91$
	$F^{\alpha t} = 0.02$	$I^{\beta t} = 0.04$

(Continued)

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* This figure is due to interpolation of logarithms and minor errors in rounding off.

Table 4. Statistical Analysis of Model I (Continued)

ANALYSIS OF VARIANCE TABLE

Source	Sum of Squares	df	Mean Square	$F_{calc.}$	$F_{tab.}$	Comment
Due to Regression	1.890	4	0.4725	215	9.15	Significant at 0.01
Deviation About Regression	0.013	6	0.0022			
Total	1.903	10				

Model II

This model has the form:

$$Y_t = K\, S^{c+\gamma t}\, I^{b+\beta t} \qquad \text{(Eq. 17)}$$

where

Y_t is shipments of warp knitted fabrics in thousands of pounds,

S is synthetic fiber production per capita,

I is disposable income per capita,

K, c, γ, b, and β, are numerical constants, and

t is time in years (with t = 0 in 1953).

The data for synthetic fiber production, as obtained from the *Textile Organon,* refers to the use of synthetic fibers in all branches of the textile industry, as described in Figure 1. A population variable for the United States was adapted so that each independent variable

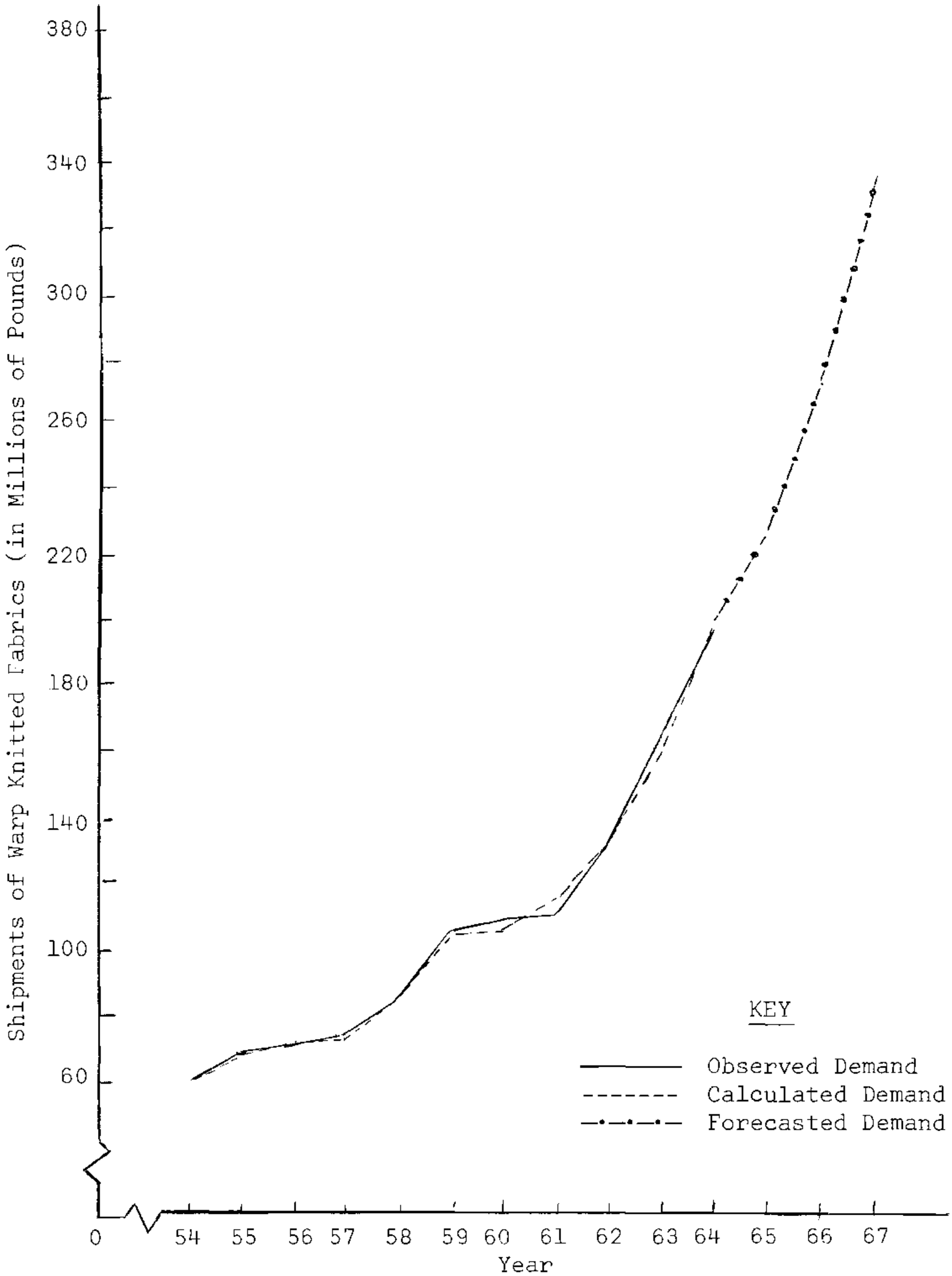

Figure 6. Observed, Calculated, and Forecasted Demand for Model I.

may be expressed as consumption per capita. Approximately 10 per cent of all synthetic fibers produced is converted into yarn to be used in the manufacture of knitted fabrics. Four per cent of synthetic fiber production is consumed by the warp knitting industry. Analysis of the econometric model developed will relate the demand for warp knitted fabrics to the ability of the consumer to purchase textile products manufactured from synthetic fibers.

Although this model has a high coefficient of multiple correlation (0.994), low standard error of the estimate (0.023) and a highly significant F ratio (from the analysis of variance table), it is not as accurate as Model I. The reduced accuracy of this model may be due to the fact that over 95 per cent of synthetic fiber production is used to manufacture other textile products. Furthermore, disposable income is spent on a variety of commodities. It should be noted that a significant F ratio implies that the variability of observed and calculated values of demand (from the model) is due to regression as opposed to deviation about the regression, which is desirable. The model developed was as follows:

$$Y_t = 2{,}531{,}000\ S^{0.118+0.052t}\ I^{-0.556-002t} \qquad \text{(Eq. 18)}$$

or

$$Y_t = 2{,}531{,}000\ \frac{S^{0.118+0.052t}}{I^{0.556+0.002t}} \qquad \text{(Eq. 19)}$$

Synthetic fiber production per capita represents the case of inelastic supply and with each succeeding year its coefficient of elasticity ($c+\gamma t$) exercises a greater positive influence over the econometric model. In 1954, $c+\gamma t$ was approximately 0.17, whereas in 1965 this exponential function increased to 0.74. It can be concluded from this model, that as synthetic fiber production increases, a greater proportion of these fibers will be used to manufacture warp knitted fabrics.

This inelastic supply effect is counter-balanced by the income elastic variable; i.e., disposable income per capita. Its coefficient of elasticity ($-b-\beta t$) is negative, and as income varies its negative magnitude increases. It should be noted that the major influence on this model is played by I^b and the minor influence by $I^{\beta t}$ (see Table 16). With reference to the independent variables selected for this model, it is expected that disposable income per capita will increase. But, the consumer can be expected to spend a greater share of his income on commodities and services other than warp knitted fabrics.

It was unusual that the partial coefficients of correlation for each independent variable were between +0.10 and -0.14, but the coefficient of multiple correlation was greater than 0.99. It can be concluded that the components of this econometric model are not closely related to the dependent variable, but the model taken as a whole closely parallels the dependent variable.

To establish forecasts for shipments of warp knitted fabrics the extrapolated values of Log S and Log I were calculated from Equations 6 and 5, respectively, and fitted into the econometric model (Eq. 18). Forecasts for S, F, and Y_t are shown in Table 5. With the expected increase in synthetic fiber production per capita, and in disposable income per capita, shipments of warp knitted fabrics should significantly increase in volume. In Model II the forecasted demand obtained for Y_t is slightly greater than demand calculated for Model I.

Table 5. Extrapolated Forecast of Demand for Model II

Year	S	I	Y_t
1965	17.53	$2,301	233,100
1966	20.35	2,382	292,360
1967	23.68	2,467	374,580

The procedure for calculating forecasted values for Y_t is shown in Table 16 in Appendix II.

Forecasts are calculated using the logarithmic extrapolations of S and I. Values of t are taken as 12, 13 and 14, for the years 1965, 1966 and 1967, respectively. The results obtained represent Log Y_{12}, Log Y_{13} and Log Y_{14}, which, when converted, result in values of Y_t in Table 5.

Statistical analysis of Model II is shown in Table 6. A graphical presentation of observed, calculated, and forecasted demand for shipments of warp knitted fabrics, using this model, may be seen in Figure 7.

Table 6. Statistical Analysis of Model II

Year	t	Observed Demand (Y)	Calculated Demand (Y_t)	Residual ($Y-Y_t$)
1954	1	60,097	60,290	- 193
1955	2	68,295	67,060	+ 1,235
1956	3	70,909	71,300	- 391
1957	4	73,005	78,700	- 5,695
1958	5	85,554	83,790	+ 1,764
1959	6	104,564	97,290	+ 7,274
1960	7	107,400	103,920	+ 3,480
1961	8	111,581	116,610	- 5,029
1962	9	135,938	140,930	- 4,992
1963	10	164,676	164,390	+ 286
1964	11	197,358	194,800	+ 2,558
				+ 297

Econometric Model $Y_t = 2{,}531{,}000\ S^{0.118+0.052t}\ I^{-0.556-0.002t}$

or

$$Y_t = 2{,}531{,}000\ \frac{S^{0.118+0.052t}}{I^{0.556+0.002t}}$$

Standard Error of the Estimate 0.02301

Coefficient of Multiple Correlation 0.9943

Coefficient of Determination 0.9885

Partial Correlation Coefficients $S^c = 0.10$ $I^b = 0.14$
$S^{\gamma t} = 0.05$ $I^{\beta t} = -0.05$

Standard Deviation of Regression Coefficients $S^c = 0.05$ $I^b = 1.60$
$S^{\gamma t} = 0.05$ $I^{\beta t} = 0.02$

ANALYSIS OF VARIANCE TABLE

Source	Sum of Squares	df	Mean Square	$F_{calc.}$	$F_{tab.}$	
Due to Regression	1.880	4	0.4700	1.35	9.15	Significant at 0.01
Deviation About Regression	0.021	6	0.0035			
Total	1.901	10				

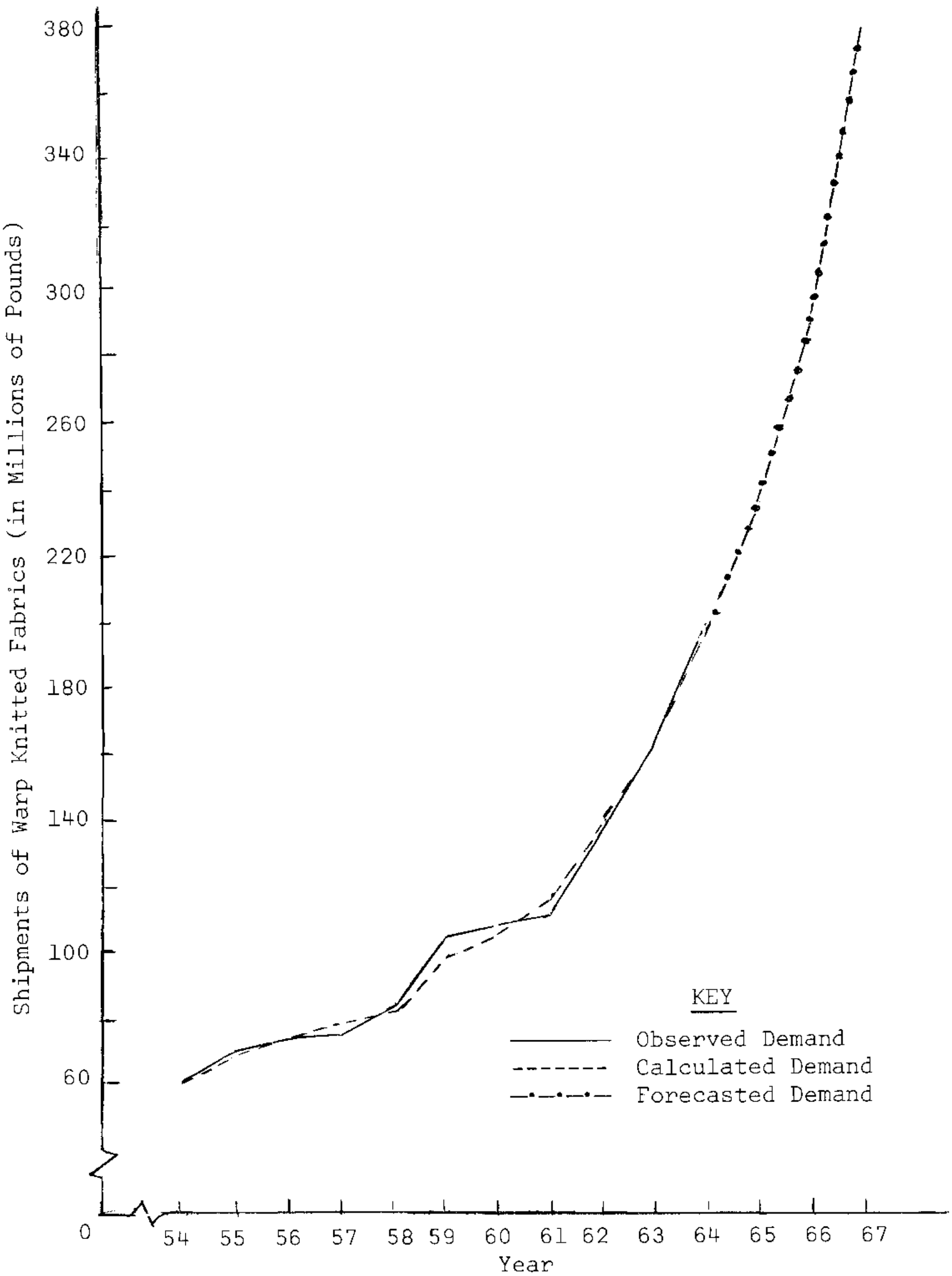

Figure 7. Observed, Calculated and Forecasted Demand for Model II.

Model III

This model has the form:

$$Y_t = K\ F^{a+\alpha t}\ I^{b+\beta t}\ S^{c+\gamma t} \qquad \text{(Eq. 20)}$$

where

Y_t is shipments of warp knitted fabrics in thousands of pounds,

F is shipments of all knitted fabrics per capita,

I is disposable income per capita,

S is synthetic fiber production per capita,

K, a, α, b, β, and γ are numerical constants, and t is time in years (with t = 0 in 1953).

It can be seen that the independent variables from the previous econometric models are incorporated in Model III. This model may be considered as a crude input-output function (see Figure 2) with production of synthetic fibers serving as the raw material, or input, necessary to manufacture a fabric, and shipments of all knitted fabrics as the output function. Disposable income serves as the purchasing medium. Analysis of this model will explain the economic forces associated with each component and their interactions. The econometric model developed has the form:

$$Y_t = 34{,}766{,}000\ F^{1.728-0.090t}\ I^{-0.779-0.027t}\ S^{-0375+0.115t} \qquad \text{(Eq. 21)}$$

or

$$Y_t = 34{,}766{,}000 \frac{F^{1.728}\ S^{0.115t}}{F^{0.090t}\ I^{0.779+0.027t}\ S^{0.375}} \qquad \text{(Eq. 22)}$$

In comparison with the previously developed models, this econometric model is statistically the most feasible as a result of a very high coefficient of multiple correlation (0.998) and a low standard error of the estimate (0.014). The F ratio (from the analysis of variance table), is highly significant, reinforcing the fact that variance is due to regression and not deviation about the regression.

The mathematical structure of this model should resolve any discrepancies seen in the previous econometric models. The partial correlation coefficients are relatively high, with shipments of all knitted fabrics having coefficients of 0.84 (F^a) and -0.55 ($F^{\alpha t}$). During the years 1959 and 1960, shipments of all knitting fabrics declined slightly (see Table 11) and although observed demand for warp knitted fabrics increased, the calculated demand declined in Model I and Model III. This fact can be attributed to the weight carried by independent variable F. The analysis of Model II did not reveal this phenomenon. A large percentage of the variability between observed and calculated demand for each model occurred during the years 1959, 1960, and 1961. This variability is due to the selection of the equation and the deviation about the regression analysis.

Disposable income per capita is an elastic function of the dependent variable. Once again, it can be seen that the consumer will spend proportionately more income on commodities and services other than warp knitted garments. Independent variables F and S have exponential function parameters; i.e., $+a-\alpha t$, and $-c+\gamma t$, containing different signs. With respect to shipments of all knitted fabrics, F, its coefficients of elasticity ($+a-\alpha t$) will decrease and eventually become negative with the passage of time. The exponential function parameters of synthetic fiber production ($-c+\gamma t$) is inelastic and with the passage of time becomes increasingly inelastic. This phenomenon can be observed by studying the calculations shown in Table 17. The statistical analysis of Model III is shown in Table 9.

Predictions for the demand for warp knitted fabrics during the years 1965, 1966, and 1967 are shown in Table 7. The extrapolated values for Log F, Log I, and Log S, used to calculate demand, are the same as calculated previously in conjunction with Model I and Model II.

Table 7. Extrapolated Forecast of Demand for Model III

Year	F	I	S	Y_t
1965	2.67	$2,301	17.53	228,430
1966	3.06	2,382	20.35	288,320
1967	3.50	2,467	23.68	368,340

A comparison of the forecasts calculated for each model is shown in Table 8. As expected, the results of Model III lie between the demand calculated for the previous two models. The close agreement of the extrapolated forecasts obtained from each model can be readily observed. Unless any unforeseen development occurs, demand for warp knitted fabrics should be in the vicinity of 230 million pounds in 1965, 285 million pounds in 1966, and 360 million pounds in 1967.

The procedure for calculating the forecasted values of Y_t is shown in Table 17.

A graphical presentation of observed, calculated, and forecasted demand for shipments of warp knitted fabrics, using this model may be seen in Figure 8.

Table 8. Comparison of Forecasted Demand (Y_t) for the Three Models

Year	Model I	Model II	Model III	Mean
1965	227,550	233,100	228,430	229,690
1966	273,520	292,360	288,320	284,730
1967	332,400	374,580	368,340	358,440

Table 9. Statistical Analysis of Model III

Year	t	Observed Demand (Y)	Calculated Demand (Y')	Residual (Y'Y')
1954	1	60,097	60,000	+ 97
1955	2	68,295	67,870	+ 425
1956	3	70,909	71,680	- 771
1957	4	73,005	73,310	- 305
1958	5	85,554	86,280	- 726
1959	6	104,564	104,280	+ 284
1960	7	107,400	102,280	+5,210
1961	8	111,581	114,750	-3,169
1962	9	135,938	139,030	-3,092
1963	10	164,676	162,870	+1,806
1964	11	197,358	196,820	+ 538
				+ 297

Econometric Model $Y_t = 34{,}766{,}000\ F^{1.728-0.090t}\ I^{-0.779-0.027t}\ S^{-0.375+0.115t}$

or

$$Y_t = 34{,}766{,}000\ \frac{F^{1.728}\ S^{0.115t}}{F^{0.090t}\ I^{0.779+0.027t}\ S^{0.375}}$$

Standard Error of Estimate 0.0139

Coefficient of Multiple Correlation 0.9986

Coefficient of Determination 0.9972

Partial Correlation Coefficients	$F^a = 0.84$	$I^b = -0.33$	$S^c = -0.38$
	$F^{\alpha t} = -0.55$	$I^{\beta t} = -0.50$	$S^{\gamma t} = 0.55$
Standard Deviation of Regression Coefficients	$F^a = 0.55$	$I^b = 1.10$	$S^c = 0.45$
	$F^{\alpha} = 0.07$	$I^{\beta t} = 0.02$	$S^{\gamma t} = 0.09$

Table 9. Statistical Analysis of Model III (Continued)

ANALYSIS OF VARIANCE

Source	Sum of Squares	df	Mean Square	$F_{Calc.}$	$F_{.01}$
Due to Regression	1.890	6	0.3150	210	15.2
Deviation About Regression	0.006	4	0.0015		
Total	1.896				

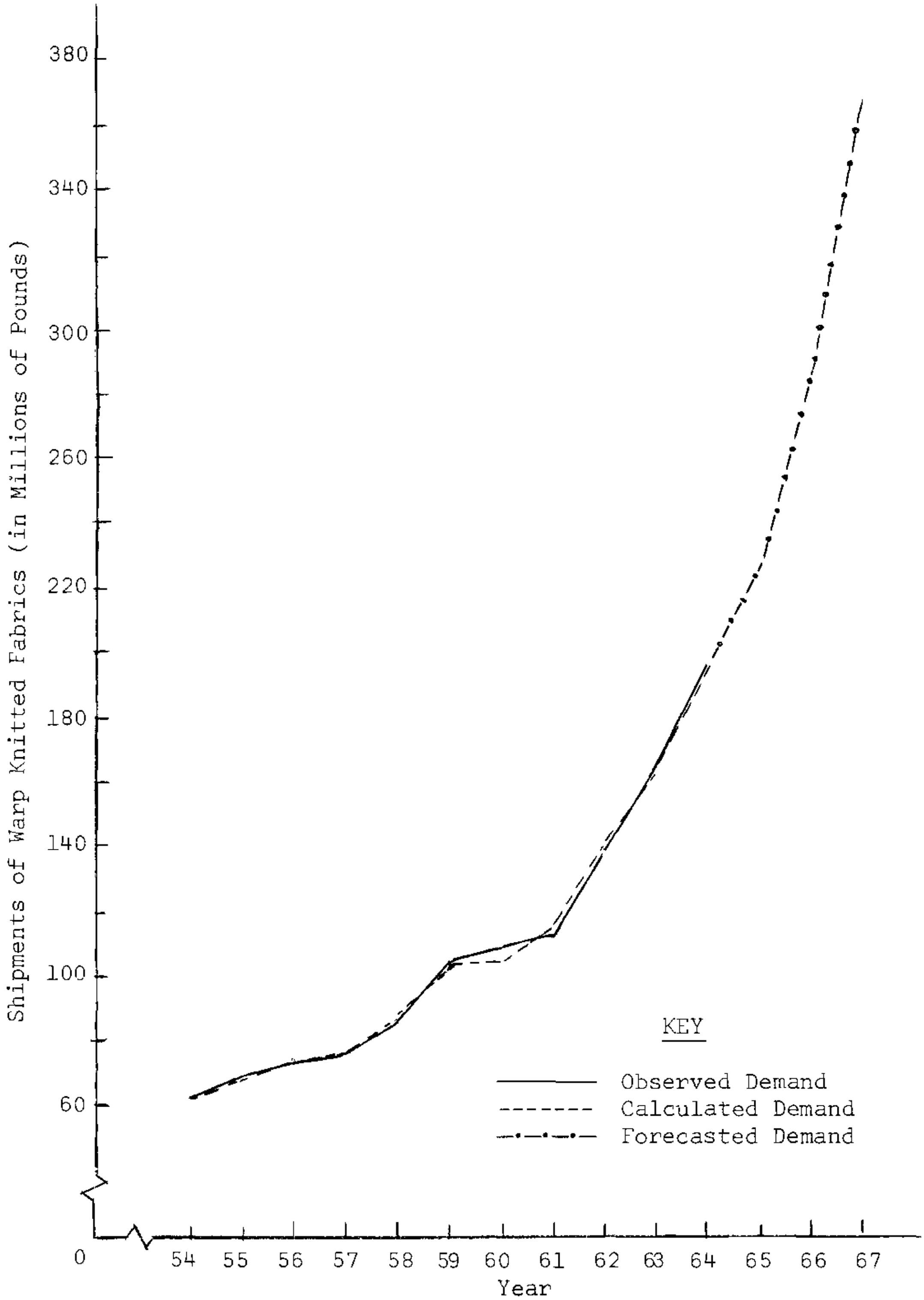

Figure 8. Observed, Calculated and Forecasted Demand Using Model III.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

The purpose of this study was to analyze and forecast demand for warp knitted fabrics using the techniques of econometric analysis and regression analysis. Tests of the three models developed yielded the following results:

1. a coefficient of multiple correlation greater than 0.99,
2. a standard error of the estimate less than 0.025,
3. a significant F ratio from the analysis of variance table, stating that variance is due to regression as opposed to deviation about the regression.

From these facts, it can be concluded that the basic equation used:

$$Y_t = K\, X_1^{a+\alpha t}\, X_2^{b+\beta t}\, X_3^{c+\gamma t} \ldots X_n^{z+\zeta t} \qquad \text{(Eq. 1)}$$

was a good choice with reference to the data available. Furthermore, this equation may be used in the analysis of any time series where the dependent variable seems to be increasing or decreasing exponentially with respect to time.

The validity of the predictions for each model will be a function of the accuracy of the extrapolation made for each independent variable. These extrapolations exhibited moderate variation among the models. As shown in Table 8, Model I resulted in the "pessimistic" forecast, Model

II resulted in the "optimistic" forecast, while Model III resulted in an "in between" forecast, which incidentally is very close to the mean of the three forecasts. For 1965 the maximum and minimum forecasts were less than 3 per cent of one another. For 1966 the forecasts were within 7 per cent of one another and for 1967 the range increased to 11 per cent.

Each econometric model predicts an increase in shipments of warp knitted fabrics. These predictions are based on the assumption that shipments of all knitted fabrics, production of synthetic fibers, and disposable income will increase. As data are obtained for each forecasted year, it is recommended that it be incorporated into the model to observe any changes in parameters. New forecasts should then be derived for a three-year period.

Data for each variable were considered on an annual basis. If monthly, quarterly or semi-annual data for shipments of warp knitted fabrics could be obtained, a short-term analysis may yield some interesting results as to seasonal fluctuations, the impact of advertising, sales campaigns, etc. The textile industry, and its subsets, has long been regarded as an industry subject to seasonal variations.

Each econometric model is only as accurate as the economic reasoning upon which it is based. An econometric model may be mathematically sound, but economically unsound. Citing the work of Silk (22), Wilson (23) states:

1. most mathematical models are static and inflexible to short-run changes,
2. mathematical precision is often applied to rough data,
3. mathematical assumptions necessary to derive regression equations do not apply in practice,

4. statisticians do not consider the element of objective judgement,
5. independent variables often cannot be predicted with any more accuracy than the dependent variable.

Frequently, certain data desired for the model are not available, is incomplete, or is from an unreliable source. The choice of a model should be selected with great care.

The econometric models used in this study were applied to a subset of the textile industry, i.e., warp knitted fabrics. If further study is to be undertaken in this area, a model involving the following variables might yield interesting results:

1. Dependent variable:
    a. shipments of all textile fabrics.
2. Independent variables:
    a. shipments of woven fabrics,
    b. shipments of knitted fabrics,
    c. shipments of all other fabrics.

The selection of the basic equation will depend upon the analysis of the data for the dependent variable. This econometric model will aid in understanding the relationship of each sub-function of the textile industry with respect to its growth potential. A model may also be developed to analyze the components of one particular subset of the textile industry; e.g., shipments of woven fabrics may be segregated as follows:

1. shipments of all cotton fabrics,
2. shipments of cotton-synthetic fabric blends; e.g., a 65 per cent dacron—35 per cent cotton fabric,

3. shipments of wool and worsted fabrics,
4. shipments of wool-synthetic fabric blends,
5. shipments of silk fabrics,
6. shipments of 100 per cent synthetic fabrics, and
7. shipments of other fabrics.

The econometric models developed in this study accomplished both objectives stated in the beginning of this report; namely, to analyze and forecast demand for shipments of warp knitted fabrics.

APPENDIX I

THE NON-ECONOMETRICIAN'S LAMENT

As soon as I could safely toddle
My parents handed me a model.
My brisk and energetic pater
Provided the accelerator,
My mother, with her kindly gumption,
The function guiding my consumption;
And every week I had from her
A lovely new parameter,
With lots of little leads and lags
In pretty parabolic bags.

With optimistic expectations
I started on my explorations,
And swore to move without a swerve
Along my sinusoidal curve.
Alas! I knew how it would end;
I've mixed the cycle and the trend,
And fear that, growing daily skinnier,
I have at length become non-linear.
I wander glumly round the house
As though I were exogeneous,
And hardly capable of feeling
The difference 'tween floor and ceiling.
I scarcely now, a pallid ghost,
Can tell *ex ante* from *ex post*;
My thoughts are sadly inelastic,
My acts incurably stochastic.

-Sir Dennis Robertson

APPENDIX II

Table 10. Ratio of Warp Knitted Shipments to All Knitting Industry Shipments

Year	A All Knitted Cloth for Sale (In 1,000 Lbs.)	B Warp Knit Cloth for Sale (In 1,000 Lbs.)	B/A
1947	118,909	36,617	.31
1948	122,168	39,354	.32
1949	147,853	42,910	.29
1950	162,803	47,208	.29
1951	148,747	43,835	.29
1952	170,518	52,694	.31
1953	164,193	47,249	.29
1954	177,155	60,097	.34
1955	197,990	68,295	.34
1956	207,400	70,909	.34
1957	212,440	73,005	.34
1958	242,452	85,554	.35
1959	280,426	104,564	.37
1960	283,104	107,400	.38
1961	316,289	111,581	.35
1962	354,016	135,938	.38
1963	400,878	164,676	.41
1964	474,245	197,358	.42

Table 11. Data for Independent Variables Used in Models

Year	Population of U.S.A. in Thousands of Persons	Disposable* Income in Millions	I Disposable Income per Capita	Synthetic Fiber Production in Thousands of Pounds	S Synthetic Fiber Production in Thousands of Pounds	Shipments of All Knitted Fabrics in Thousands of Pounds	F Consumption of Knitted Fabrics per Capita
1954	163,026	$256,900	$1,576	1,429,500	8.77	177,155	1.09
1955	165,931	274,400	1,654	1,715,800	10.34	197,990	1.19
1956	168,903	292,900	1,734	1,644,700	9.74	207,400	1.23
1957	171,984	308,800	1,796	1,764,700	10.26	212,440	1.24
1958	174,882	318,000	1,818	1,629,200	9.32	242,542	1.39
1959	177,830	337,100	1,897	1,959,500	11.02	280,426	1.58
1960	180,684	349,900	1,936	1,884,900	10.43	283,104	1.57
1961	183,756	364,700	1,985	1,995,400	10.86	316,289	1.72
1962	186,656	384,600	2,060	2,435,300	13.05	354,016	1.90
1963	189,375	402,500	2,125	2,696,700	14.24	400,878	2.12
1964	192,116	431,800	2,248	3,078,000	16.02	474,245	2.47

Table 12. Parabolic Extrapolation for Log F

Year	t	Log F	t^2	t Log F	t^2 Log F	t^4	Calc. Log F	Resid.	Calc. F
1954	-5	0.03607	25	-0.18035	0.90175	625	.04382	-.00775	1.11
1955	-4	0.07361	16	-0.29444	1.11776	256	.06254	+.01107	1.16
1956	-3	0.08920	9	-0.26760	0.80280	81	.08546	+.00374	1.22
1957	-2	0.09174	4	-0.18348	0.36696	16	.10988	-.01814	1.29
1958	-1	0.14189	1	-0.14189	0.14189	1	.13850	+.00339	1.38
1959	0	0.19783	0	0	0	0	.17042	+.02741	1.48
1960	1	0.19502	1	0.19502	0.19502	1	.20564	-.01062	1.61
1961	2	0.23583	4	0.47166	0.94332	16	.24416	-.00833	1.75
1962	3	0.27775	9	0.83325	2.49975	81	.28598	-.00823	1.93
1963	4	0.32464	16	1.29856	5.19424	256	.33110	-.00646	2.14
1964	5	0.39243	25	1.96215	9.81075	625	.37952	+.09291	2.40
Total		2.05596	110	3.69288	21.97424	1,958	2.05697	-.00101	

Extrapolating Equation:

$$\text{Log } F_t = 0.17042 + 0.0336t + 0.0017t^2$$

Forecasts for F in Pounds per Capita

Year	t	Log F	F
1965	6	0.43124	2.699
1966	7	0.48626	3.064
1967	8	0.54458	3.504

Table 13. Parabolic Extrapolation for Log I

Year	t	Log I	t^2	t Log I	t^2 Log I	t^4	Calc. Log I	Resid.	Calc. I
1954	-5	3.19750	25	-15.98750	79.93750	625	3.20415	-.00665	1,600
1955	-4	3.21849	16	-12.87396	51.49584	256	3.21789	+.00060	1,652
1956	-3	3.23907	9	- 9.69291	29.15163	81	3.23176	+.00731	1,705
1957	-2	3.25419	4	- 6.50838	13.01676	16	3.24573	+.00846	1,761
1958	-1	3.25942	1	- 3.25942	3.25942	1	3.26038	-.00096	1,821
1959	0	3.27814	0	0	0	0	3.27406	+.00408	1,880
1960	1	3.28702	1	3.28702	3.28702	1	3.28892	-.00190	1,945
1961	2	3.29791	4	6.59582	13.19164	16	3.30285	-.00494	2,008
1962	3	3.31398	9	9.94194	29.82582	81	3.31744	-.00346	2,077
1963	4	3.32742	16	13.30968	53.23872	250	3.33243	-.00501	2,150
1964	5	3.35172	25	16.75860	83.79300	625	3.34695	+.00477	2,223
Total		36.02486	110	1.57089	360.19745	1,958	36.02256	+.00230	

Extrapolating Equation:

$$\text{Log } I_t = 3.27406 + 0.0143t + 0.0001t^2$$

Year	t	Log I	I
1965	6	3.36189	\$2,301
1966	7	3.37694	2,382
1967	8	3.39211	2,467

Table 14. Parabolic Extrapolation for Log S

Year	t	Log S	t^2	t Log S	t^2 Log S	t^4	Calc. Log S	Resid.	Calc. S
1954	-5	.94292	25	-4.71460	23.59800	625	.98322	-0.04030	9.62
1955	-4	1.01452	16	-4.05808	16.23232	256	.97478	+0.03974	9.50
1956	-3	.98845	9	-2.96535	8.89605	81	.97824	+ .01021	9.51
1957	-2	1.01119	4	-2.02238	4.04476	16	.98467	+ .02652	9.65
1958	-1	.96923	1	- .9623	.96923	1	.99705	- .02782	9.93
1959	0	1.04215	0	0	0	0	1.01538	+ .02677	10.32
1960	1	1.01838	1	1.01838	1.01838	1	1.03965	- .02127	10.96
1961	2	1.03580	4	2.07160	4.14320	16	1.06989	- .03409	11.75
1962	3	1.111550	9	3.34650	10.03950	81	1.10606	+ .00944	12.77
1963	4	1.15351	16	4.61404	18.45616	256	1.14530	+ .00821	13.97
1964	5	1.20468	25	6.02340	30.11700	625	1.19626	+ .00842	15.71
Total		11.49633	110	2.34338	117.51460	1,958	11.49050	+ .00583	

Extrapolating Equation:

$$\text{Log } S_t = 1.01538 + 0.0213t + 0.0030t^2$$

Forecasts for S in Pounds per Capita

Year	t	Log S	S
1965	6	1.24378	17.530
1966	7	1.30844	20.345
1967	8	1.37436	23.679

Table 15. Calculation of Extrapolated Forecast for Model I

Econometric* Model $Y_t = 501.800\ F^{1.386+0.001t}\ I^{0.008+0.001t}$

or

$$\text{Log } Y_t = \text{Log } 4.70089 + 1.386 \text{ Log } F + 0.001\ (t) \text{ Log } F + 0.008 \text{ Log } I + 0.001\ (t) \text{ Log } I$$

FORECAST FOR 1965

Log K		4.70089
a Log F	(1.386)(0.43124)	.59770
α t Log F	(0.001)(12)(0.43124)	.00517
b Log I	(0.008)(3.36189)	.02617
β t Log I	(0.001)(12)(3.36189)	.02715
Total		5.35708
Conversion		227,550

FORECAST FOR 1966

Log K		4.70089
a Log F	(1.386)(0.48626)	.67396
α t Log F	(0.001)(13)(0.48626)	.00632
b Log I	(0.008)(3.37694)	.02629
β t Log I	(0.001)(13)(3.37694)	.02954
Total		5.43700
Conversion		273,520

FORECAST FOR 1967

Log K		4.70089
a Log F	(1.386)(0.54458)	.75479
α t Log F	(0.001)(14)(0.54458)	.00762
b Log I	(0.008)(3.39211)	.02640
β t Log I	(0.001)(14)(3.39211)	.03196
Total		5.52166
Conversion		332,400

* Numerical constants in this model are rounded off, if taken to six decimal places they would be

a = 1.385776
α = 0.000957
b = 0.007784
β = 0.000673

Table 16. Calculation of Extrapolated Forecast for Model II

Econometric* Model $Y_t = 2{,}531{,}000\ S^{0.118+0.052t}\ I^{-0.556-0.002t}$

or

$$\text{Log } Y_t = \text{Log } 6.40452 + 0.118 \text{ Log } S + 0.052\ (t) \text{ Log } S - 0.556 \text{ Log } I - 0.002\ (t) \text{ Log } I$$

FORECAST FOR 1965

Log K		6.40452
c Log S	(0.118)(1.24378)	.14677
γ t Log S	(0.052)(12)(1.24378)	.77015
b Log I	(-0.556)(3.36141)	-1.86851
β t Log I	(-0.002)(12)(3.36141)	- .08538
Total		5.36755
Conversion		233,100

FORECAST FOR 1966

Log K		6.40452
c Log S	(0.118)(1.30844)	.15440
γ t Log S	(0.052)(13)(1.30844)	.87702
b Log I	(-0.556)(3.37694)	-1.87714
β t Log I	(-0.002)(13)(3.37694)	- .09287
Total		5.46593
Conversion		292,360

FORECAST FOR 1967

Log K		6.40452
c Log S	(0.118)(1.37436)	.16217
γ t Log L	(0.052)(14)(1.37436)	.99284
b Log I	(-0.556)(3.39211)	-1.88577
β t Log I	(-0.002)(14)(3.39211)	- .10041
Total		5.57335
Conversion		374,580

* Numerical constants in this model are rounded off, if taken to six decimals places they would be

$c = 0.118002$
$\gamma = 0.051606$
$b = -0.555869$
$\beta = -0.002116$

Table 17. Calculation of Extrapolated Forecast for Model III

Econometric* Model $Y_t = 34,766,000\ F^{1.728-0.90t}\ I^{-0.779-0.027t}\ S^{-0.375+0.115t}$

or

$\text{Log } Y_t = \text{Log } 7.54115 - 1.728 \text{ Log } F - 0.090\ (t) \text{ Log } F - 0.779 \text{ Log } I - 0.027\ (t) \text{ Log } I - 0.375 \text{ Log } S + 0.115\ (t) \text{ Log } S$

FORECAST FOR 1965

Log K		7.54115
a Log F	(1.728)(0.43124)	.74543
α t Log F	(-0.090)(12)(0.43124)	- .46367
b Log I	(-0.779)(3.36189)	-2.61818
β t Log I	(-0.027)(12)(3.36189)	-1.09597
c Log S	(-0.375)(1.24378)	- .46642
γ t Log S	(0.115)(12)(1.24378)	1.71642
Total		5.35876
Conversion		228,430

FORECAST FOR 1966

Log K		7.54115
a Log F	(1.728)(0.48626)	.84051
α t Log F	(-0.090)(13)(0.48626)	- .56640
b Log I	(-0.779)(3.37694)	-2.62963
β t Log I	(-0.027)(13)(3.37694)	-1.19375
c Log S	(-0.375)(1.30844)	- .49067
γ t Log S	(0.115)(13)(1.30844)	1.95873
Total		5.45988
Conversion		288,320

FORECAST FOR 1967

Log K		7.54115
a Log F	(1.728)(0.54458)	.94135
α t Log F	(-0.090)(14)(0.54458)	- .68312
b Log I	(-0.779)(3.39211)	-2.64171
β t Log F	(-0.027)(14)(3.39211)	-1.29138
c Log S	(-0.375)(1.37436)	- .51539
γ t Log S	(0.115)(14)(1.37436)	2.21547
Total		5.56637
Conversion		368,440

* Numerical constants in this model are rounded off. If taken to six decimal places they would be

a = 1.728582
α = -0.089615
b = -0.778782
β = -0.027191
c = 0.375004
γ = 0.115151

APPENDIX III

```
          BEGIN
COMMENT        MULTIPLE REGRESSION AND CORRELATION ANALYSIS
               CHARLES L. CLARK
               PROFESSIONAL SERVICES DIVISIONAL GROUP
               BURROUGHS CORPORATION
               PROGRAM CONTAINS 416 CARDS
               FIRST RELEASE DATE 12-1-64  ;
INTEGER        H, I, M, N, Q ;
INTEGER ARRAY  ID[0:4] ;
BOOLEAN        TR,MEAN,VAR,NORM,INV,RES,PCC,CC,CCINV,SWICH ;
LABEL          START, LL1,NOMO ;
FORMAT IN      FORM1 (5A6, 4I5, 6L5/3L5) ,
               FORM2 (26I3) ,
               FORM3 (5E13.6) ;
FORMAT OUT     TITLE(X34,"GENERAL MULTIPLE REGRESSION AND ",
                     "CORRELATION ANALYSIS"///X45,5A6//
                     X1,"SAMPLE SIZE",X2,I5/
                     X1,"NO. OF INDEPENDENT VARIABLES",X2,I3/
                     X1,"NO. OF DEPENDENT VARIABLES",X4,I3//) ,
               FRM1 (X1,"NORMAL MATRIX"/) ,
               FRM2 (X1,"ROW",I3/(6(X1,I3,X1,E15.8)/)) ,
               FRM3 (//X1,"MEANS OF INDEPENDENT AND " ,
                     "DEPENDENT VARIABLES"/) ,
               FRM4 ((6(X1,I3,X1,E15.8)/)/) ,
               FRM5 (/X1,"VARIANCES OF INDEPENDENT AND " ,
                     "DEPENDENT VARIABLES"/) ,
               FRM6 (/X1,"THE INVERSE MATRIX"/) ,
               FRM7 (/X1,"PRODUCT OF NORMAL AND INVERSE " ,
                     "MATRICES"/) ,
               FRM8 (/X47,"DEPENDENT VARIABLE NO.",I3/) ,
               FRM9 (X1,"REGRESSION COEFFICIENTS"//
                     X1,"THE CONSTANT TERM IS",X3,E15.8) ,
               FRM10(/X1,"THE ERROR SUM SQUARE IS",X3,E15.8//
                     X1,"THE ERROR MEAN SQUARE IS",X2,E15.8//
                     X1,"THE STANDARD ERROR OF ESTIMATE IS" ,
                     X3,E15.8//
```

```
                    X1,"THE COEFFICIENT OF DETERMINATION " ,
                    "IS",X6,E15.8//
                    X1,"THE MULTIPLE CORRELATION " ,
                    "COEFFICIENT IS",X2,E15.8//) ,
                FRM11(X1,"STANDARD DEVIATIONS OF THE " ,
                    "REGRESSION COEFFICIENTS"/) ,
                FRM12(/X1,"T VALUE OF REGRESSION " ,
                    "COEFFICIENTS"/) ,
                FRM13(/X36,"ANALYSIS OF VARIANCE"/X29,
                    "FOR THE MULTIPLE LINEAR REGRESSION"//
                    X7,"SOURCE OF VARIATION",X7,"D.F.",X8,
                   "SUM OF",X13,"MEAN",X15,"F"/X45,
                    "SQUARES",X11,"SQUARES",X11,"RATIO"/
                    X1,"DUE TO REGRESSION",X14,I5,X3,E15.8,
                    X3,E15.8,X3,E15.8/
                    X1,"DEVIATION ABOUT REGRESSION",X5,I5,
                    X3,E15.8,X3,E15.8/
                    X22,"TOTAL",X5,I5,X3,E15.8//) ,
                FRM14(/X51,"TABLE OF RESIDUALS"//
                    X25,"OBSERVATION",X8,"Y CALCULATED",X9,
                    "Y OBSERVED",X11,"RESIDUAL") ,
                FRM15(X28,I5,X9,E15.8,X5,E15.8,X5,E15.8) ,
                FRM16(//X25,"TEST OF EXTREME RESIDUALS"//
                    X25,"RANGE OF RESIDUALS",X20,E15.8/
                    X25,"RANGE / STANDARD ERROR OF " ,
                    "ESTIMATE",X4,E15.8//) ,
                FRM17(X1,"CORRELATION COEFFICIENTS"/) ,
                FRM18(/X1,"INVERSE OF CORRELATION MATRIX"/) ,
                FRM19(/X1,"PARTIAL CORRELATION COEFFICIENTS"/) ;
FILE IN         CARDR(1,10) ;
FILE OUT        PRINT 4 (1,15) ;
START:      BEGIN
LIST            HEAD(FOR I ← 0 STEP 1 UNTIL 4 DO ID[I],M,N,Q,
                   H,TR,MEAN,VAR,NORM,INV,RES,PCC,CC,CCINV);
                READ (CARDR,FORM1,HEAD) [NOMO] ;
            END ;
```

```
            SWICH ← FALSE ;
LL1:     BEGIN
INTEGER       J, K, L, P ;
ARRAY         X[0:Q],YSQ[0:Q-N],A[0:N+1,0:Q],B[0:H],SB[0:N] ,
              T[0:Q] ;
REAL          C, RSS, RMS, F, ERS, EMS, SE, CO, MCC, HI, LO, R, S ;
LABEL         RERED, RESDU ;
LIST          TRAN (FOR J ← 1 STEP 1 UNTIL Q DO T[J]) ,
              DATA (FOR I ← 1 STEP 1 UNTIL H DO B[I]) ,
              OUT1 (FOR J ← 0 STEP 1 UNTIL 4 DO ID[J], M, N,
                    L) ,
              OUT2 (I, FOR J ← I STEP 1 UNTIL N DO
                    [J, A[I,J]]) ,
              OUT3 (FOR I ← 1 STEP 1 UNTIL N DO [I, B[I]]) ,
              OUT4 (FOR I ← 1 STEP 1 UNTIL N DO [I, SB[I]]),
              OUT5 (FOR I ← 1 STEP 1 UNTIL N DO [I, X[I]]) ,
              OUT6 (I, FOR J ← I STEP 1 UNTIL N DO
                    [J, X[J]]) ;
PROCEDURE     INVERT (N, A) ;
VALUE         N ;
INTEGER       N ;
ARRAY         A[0,0] ;
COMMENT       THIS PROCEDURE INVERTS A SYMMETRIC MATRIX
              A[I,J] OF WHICH THE UPPER HALF IS GIVEN ;
         BEGIN
INTEGER       I, J, K, L ;
ARRAY         Z[0:N] ;
REAL          DG, R ;
              FOR K ← 1 STEP 1 UNTIL N-1 DO
         BEGIN
              DG ← A[K,K] ;
              FOR I ← K+1 STEP 1 UNTIL N DO
         BEGIN
              R ← A[K,I] / DG ;
         FOR J ← I STEP 1 UNTIL N DO
              A[I,J] ← A[I,J] - A[K,J] × R ;
```

```
                FOR L ← 1 STEP 1 UNTIL K-1 DO
                A[L,I] ← A[L,I] - A[L,K] × R ;
                A[K,I] ← -R
           END
           END ;
COMMENT         BACKWARD PASS ;
                FOR K ← 1 STEP 1 UNTIL N DO
           BEGIN
                FOR J ← K+1 STEP 1 UNTIL N DO
                Z[J] ← A[K,J] / A[J,J] ;
                Z[K] ← 1 / A[K,K] ;
                FOR I ← K STEP 1 UNTIL N DO
           BEGIN
           FUR J ← I+1 STEP 1 UNTIL N DO
                Z[I] ← Z[I] + Z[J] × A[I,J] ;
                A[K,I] ← Z[I]
           END
           END
           END  INVERT ;
LABEL           NAXT, L1, L2, L3, L4, L5, L6, L7, L8, L9, L10,
                L11, L12, L13, L14, L15 ;
SWITCH          U ← L1,L2,L3,L4,L5,L6,L7,L8,L9,L10,L11,L12,L13,L14,L1  ;
FILE            TAPEA (2,Q+1) ;
                IF TR THEN
                READ (CARDR,FORM2,TRAN) ;
                FOR I ← 0 STEP 1 UNTIL N DO
                FOR J ← I STEP 1 UNTIL Q DO
                A[I,J] ← 0 ;
                L ← Q-N ;
                FOR P ← 1 STEP 1 UNTIL L DO
                YSQ[P] ← 0 ;
                FOR L ← 1 STEP 1 UNTIL M DO
           BEGIN
                READ (CARDR,FORM3,DATA) ;
                FOR J ← 1 STEP 1 UNTIL Q DO
           BEGIN
```

```
                IF TR THEN
          BEGIN
                GO TO U[T[J]] ;
L1:             X[J] ← B[J] ;
                GO TO NAXT ;
L2:             X[J] ← B[J] × B[J] ;
                GO TO NAXT ;
L3:             X[J] ← 1 / B[J] ;
                GO TO NAXT ;
L4:             X[J] ← SQRT (B[J]) ;
                GO TO NAXT ;
L5:             X[J] ← EXP (B[J]) ;
                GO TO NAXT ;
L6:             X[J] ← LN (B[J]) ;
                GO TO NAXT ;
L7:             X[J] ← SIN (B[J]) ;
                GO TO NAXT ;
L8:             X[J] ← COS (B[J]) ;
                GO TO NAXT ;
L9:             X[J] ← B[J+1] ;
                GO TO NAXT ;
L10:            X[J] ← B[J+2] ;
                GO TO NAXT ;
L11:            X[J] ← B[J+3] ;
                GO TO NAXT ;
L12:            X[J] ← B[J+4] ;
                GO TO NAXT ;
L13:            X[J] ← B[J+5] ;
                GO TO NAXT ;
L14:            X[J] ← B[J+6] ;
                GO TO NAXT ;
L15:            X[J] ← B[J+7] ;
                GO TO NAXT ;
NAXT:     END
                ELSE
                X[J] ← B[J]
```

```
        END ;
            WRITE(TAPEA,Q+1,X[*])
        END;
            REWIND(TAPEA) ;
RERED:      FOR L ← 1 STEP 1 UNTIL M DO
        BEGIN
            READ (TAPEA,Q+1,X[*]) ;
            IF SWICH THEN
        BEGIN
            F ← B[0] ;
            FOR I ← 1 STEP 1 UNTIL N DO
            F ← F + B[I] × X[I] ;
            C ← F - X[P] ;
            WRITE (PRINT,FRM15,L,F,X[P],C) ;
            IF HI < C THEN
            HI ← C ;
            IF LO > C THEN
            LO ← C
        END
            ELSE
        BEGIN
            FOR K ← 1 STEP 1 UNTIL Q DO
        BEGIN
            C ← X[K] ;
            A[0,K] ← A[0,K] + C ;
            IF K ≤ N THEN
        BEGIN
            FOR J ← K STEP 1 UNTIL Q DO
            A[K,J] ← A[K,J] + C × X[J]
        END
        END ;
            FOR J ← N+1 STEP 1 UNTIL Q DO
            YSQ[J-N] ← YSQ[J-N] + X[J] × X[J]
        END
        END ;
            IF SWICH THEN
```

```
             GO TO RESDU ;
             FOR I ← 1 STEP 1 UNTIL Q DO
        BEGIN
             R ← A[0,I] / M ;
             IF I ≤ N THEN
        BEGIN
             C ← A[I,I] - A[0,I] × R ;
             A[I,I] ← C ;
             A[I,0] ← C ;
             FOR J ← I+1 STEP 1 UNTIL N DO
        BEGIN
             C ← A[I,J] - A[0,J]×R ;
             A[I,J] ← C ;
             A[J,I] ← C
        END ;
             FOR J ← N+1 STEP 1 UNTIL Q DO
             A[I,J] ← A[I,J] - A[0,J]×R
        END
             ELSE
             YSQ[I-N] ← YSQ[I-N] - A[0,I] × R ;
             A[0,I] ← R
        END ;
             L ← Q-N ;
             WRITE(PRINT[PAGE]) ;
             WRITE(PRINT,TITLE,OUT1) ;
COMMENT      PRINTOUT OF NORMAL MATRIX ;
             IF NORM THEN
        BEGIN
             WRITE (PRINT,FRM1) ;
             FOR I ← 1 STEP 1 UNTIL N DO
             WRITE (PRINT,FRM2,OUT2)
        END ;
COMMENT      COMPUTATION OF MEANS AND VARIANCES ;
             IF MEAN THEN
        BEGIN
             FOR I ← 1 STEP 1 UNTIL Q DO
```

```
                X[I] ← A[0,I] ;
                WRITE (PRINT,FRM3) ;
                L ← N ;
                N ← Q ;
                WRITE (PRINT,FRM4,OUT5) ;
                N ← L
            END ;
                IF VAR THEN
            BEGIN
                FOR I ← 1 STEP 1 UNTIL N DO
                X[I] ← A[I,0] / (M-1) ;
                FOR I ← N+1 STEP 1 UNTIL Q DO
                X[I] ← YSQ[I-N] / (M-1) ;
                WRITE (PRINT,FRM5) ;
                L ← N ;
                N ← Q ;
                WRITE (PRINT,FRM4,OUT5) ;
                N ← L
            END ;
                INVERT (N, A) ;
COMMENT         PRINTOUT OF INVERSE MATRIX ;
                IF INV THEN
            BEGIN
                WRITE (PRINT,FRM6) ;
                FOR I ← 1 STEP 1 UNTIL N DO
                WRITE (PRINT,FRM2,OUT2) ;
COMMENT         PRODUCT OF NORMAL AND INVERSE MATRICES ;
                WRITE (PRINT,FRM7) ;
                FOR I ← 1 STEP 1 UNTIL N DO
            BEGIN
                FOR J ← I STEP 1 UNTIL N DO
            BEGIN
                X[J] ← 0 ;
                FOR L ← 1 STEP 1 UNTIL I-1 DO
                X[J] ← X[J] + A[L,I] × A[J,L] ;
                FOR L ← I STEP 1 UNTIL N DO
```

```
        BEGIN
             IF L < J THEN
             S ← A[J,L]
             ELSE IF L > J THEN
             S ← A[L,J]
             ELSE
             S ← A[J,0] ;
             X[J] ← X[J] + S × A[I,L]
        END
        END ;
             WRITE (PRINT,FRM2,OUT6)
        END
        END ;
             FOR I ← 1 STEP 1 UNTIL N DO
        BEGIN
             R ← A[I,0] ;
             FOR J ← I+1 STEP 1 UNTIL N DO
             A[J,I] ← A[J,I] / SQRT (R × A[J,0])
        END ;
COMMENT      CONSIDER EACH DEPENDENT VARIABLE SEPARATELY ;
             FOR P ← N+1 STEP 1 UNTIL Q DO
        BEGIN
COMMENT      COMPUTATION OF REGRESSION COEFFICIENTS ;
             FOR I ← 1 STEP 1 UNTIL N DO
        BEGIN
             B[I] ← 0 ;
             FOR L ← 1 STEP 1 UNTIL I-1 DO
             B[I] ← B[I] + A[L,I] × A[L,P] ;
             FOR L ← I STEP 1 UNTIL N DO
             B[I] ← B[I] + A[I,L] × A[L,P]
        END ;
             B[0] ← A[0,P] ;
             FOR I ← 1 STEP 1 UNTIL N DO
             B[0] ← B[0] - B[I] × A[0,I] ;
COMMENT      COMPUTATION OF REGRESSION SUM OF SQUARES ;
             RSS ← 0 ;
```

```
            FOR I ← 1 STEP 1 UNTIL N DO
            RSS ← RSS + B[I] × A[I,P] ;
            RMS ← RSS / N ;
COMMENT     COMPUTATION OF ERROR SUM SQUARE ;
            C ← YSQ[P-N] ;
            ERS ← C - RSS ;
COMMENT     COMPUTATION OF ERROR MEAN SQUARE AND
            STANDARD ERROR OF ESTIMATE ;
            K ← M-1 ;
            L ← K-N ;
            EMS ← ERS / L ;
            SE ← SQRT (EMS) ;
COMMENT     COMPUTATION OF COEFFICIENT OF DETERMINATION ;
            CD ← RSS / C ;
COMMENT     MULTIPLE CORRELATION COEFFICIENT ;
            MCC ← SQRT (CD) ;
COMMENT     COMPUTATION OF STANDARD DEVIATIONS AND T
            VALUES OF REGRESSION COEFFICIENTS ;
            FOR I ← 1 STEP 1 UNTIL N DO
        BEGIN
            S ← SQRT (EMS × A[I,I]) ;
            SB[I] ← S ;
            X[I] ← B[I] / S
        END ;
            I ← P-N ;
            WRITE (PRINT,FRM8,I) ;
            WRITE (PRINT,FRM9,B[0]) ;
            WRITE (PRINT,FRM4,OUT3) ;
            WRITE (PRINT,FRM10,ERS,EMS,SE,CD,MCC) ;
            WRITE (PRINT,FRM11) ;
            WRITE (PRINT,FRM4,OUT4) ;
            WRITE (PRINT,FRM12) ;
            WRITE (PRINT,FRM4,OUT5) ;
COMMENT     ANALYSIS OF VARIANCE ;
            F ← RMS / EMS ;
            WRITE (PRINT,FRM13,N,RSS,RMS,F,L,ERS,EMS,K,C);
```

```
COMMENT      COMPUTATION OF RESIDUES ;
             IF RES THEN
        BEGIN
             REWIND(TAPEA) ;
             SWICH ← TRUE ;
             WRITE (PRINT,FRM14) ;
             HI ← -0.9@+49 ;
             LO ←  0.9@+49 ;
             GO TO RERED ;
RESDU:       CD ← HI - LO ;
             F ← CD / SE ;
             WRITE (PRINT,FRM16,CD,F)
        END
        END ;
             IF PCC THEN
        BEGIN
COMMENT      CONSIDER EACH DEPENDENT VARIABLE SEPARATELY ;
             FOR P ← N+1 STEP 1 UNTIL Q DO
        BEGIN
COMMENT      GET AUGMENTED CORRELATION COEFFICIENT MATRIX ;
             FOR I ← 1 STEP 1 UNTIL N+1 DO
             A[I,I] ← 1 ;
             FOR K ← 1 STEP 1 UNTIL N DO
        BEGIN
             FOR J ← K+1 STEP 1 UNTIL N DO
             A[K,J] ← A[J,K] ;
             A[K,N+1] ← A[K,N+1] / SQRT (A[K,0] × YSQ[1])
        END ;
             L ← P-N ;
             WRITE (PRINT,FRM8,L) ;
             N ← N+1 ;
             IF CC THEN
        BEGIN
             WRITE (PRINT,FRM17) ;
             FOR I ← 1 STEP 1 UNTIL N DO
             WRITE (PRINT,FRM2,OUT2)
```

```
        END ;
COMMENT       INVERT AUGMENTED CORRELATION MATRIX ;
              INVERT (N, A) ;
              IF CCINV THEN
        BEGIN
              WRITE (PRINT,FRM18) ;
              FOR I ← 1 STEP 1 UNTIL N DO
              WRITE (PRINT,FRM2,OUT2)
        END ;
COMMENT       DETERMINE PARTIAL CORRELATION COEFFICIENTS ;
              R ← A[N,N] ;
              N ← N-1 ;
              FOR J ← 1 STEP 1 UNTIL N DO
              X[J] ← -A[J,N+1] / SQRT (R × A[J,J]) ;
              WRITE (PRINT,FRM19) ;
              WRITE (PRINT,FRM4,OUT5) ;
COMMENT       MOVE COLUMN FOR NEXT DEPENDENT VARIABLE ;
              IF P+1 ≤ Q THEN
        BEGIN
              FOR J ← I STEP 1 UNTIL N+1 DO
              A[J,N+1] ← A[J,P+1]
        END
        END
        END
        END ;
              GO TO START ;
NOMO:   END   PROGRAM.
```

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